Documenting the Post-2000 Decline in the Idiosyncratic Volatility Effect

Chaehyun Pyun*
ch.pyun@uga.edu

Terry College of Business
University of Georgia
620 South Lumpkin Street
Athens, GA 30602
USA

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Abstract

My paper investigates in which periods the idiosyncratic volatility (IVOL) anomaly is observable, and the trend in recent years. It uses a graphical methodology that allows the reader to assess the effects of different starting and ending months. Plots for the value-weighted portfolio show that near the end of the sample period, the Ang, Hodrick, Xing, and Zhang (2006) anomaly either attenuates or disappears. Consistent with Bali and Cakici (2008), the effect is weaker and insignificant for the equal-weighted portfolio. Using 5F and 6F benchmark return models shows similar results that differ quantitatively, but not qualitatively.

Keywords: idiosyncratic volatility, factor models, graphical diagnostic
JEL Classification: G11, G12, G14

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In an influential paper, Ang, Hodrick, Xing, and Zhang (2006) show that idiosyncratic volatility (IVOL) estimated using daily data is negatively correlated with expected returns in the cross-section. This contradicts the results of earlier empirical investigations and classical theories. Bali and Cakici (2008) show that the weighting scheme used to form the zero-investment portfolio plays a role, too. The IVOL anomaly appears when the portfolio is value-weighted, not equal-weighted. They further find that the anomaly disappears when different breakpoints are used. Therefore, they conclude that the relation is not robust.

In my paper, using an updated sample period from 1963 to 2018, I investigate both in which periods the IVOL anomaly is observable, and the trend in recent years. The method is graphical, extending a similar diagnostic in Welch and Goyal (2008). The plots allow the reader to assess the effects of different starting and ending months. Moreover, my paper does so for the value-weighted portfolios of Ang et al. (2006) and for the equal-weighted portfolios of Bali and Cakici (2008).

My paper also investigates the effects of different benchmark return models. The results differ quantitatively, but not qualitatively: the equal-weighted portfolios never have significant alphas, only the value-weighted portfolios do. Somewhat surprisingly, the out-of-sample performance of the value-weighted portfolios is even better than their in-sample performance.

The paper proceeds as follows. The next section briefly reviews the data and the empirical methodology I use, which closely follows Ang et al. (2006). Section 2 explains the graphical diagnostic of the IVOL anomaly. Section 3 shows the empirical performance of the IVOL anomaly and the key findings of my paper. Section 4 summarizes the findings of my paper.

1. Data and IVOL Portfolio Construction

My paper follows the data construction and methodology of Ang et al. (2006). The daily and monthly individual stock return data are from CRSP. The factors for the regressions are downloaded from Kenneth R. French's data library.

To estimate IVOL for an individual stock, I run the three-factor Fama-French (1993) regression for each stock i

$$R_{i,t} - r_{f,t} = \alpha_i + \beta_{MKT,i}\left(R_{MKT,t} - r_{f,t}\right) + \beta_{SMB,i}SMB_t + \beta_{HML,i}HML_t + \varepsilon_{i,t}, \quad (1)$$

where $R_{i,t}$ is the return on stock i, $R_{MKT,t}$ is the market return, and $r_{f,t}$ is the risk-free rate. For each stock i, following Bali and Cakici (2008) and Ang et al. (2006), I use within-month daily return data in equation (1) to measure IVOL, defined as $\sqrt{\text{var}(\varepsilon_{i,t})} = s.d_t(\varepsilon_i)$.

For every month from July 1963 to December 2018 (666 months), I then form value-weighted and equal-weighted quintile portfolios by sorting the stocks based on the calculated in-time lagged IVOL. Portfolio "LV" is the portfolio of stocks with the lowest IVOL, and portfolio "HV" is the portfolio of stocks with the highest IVOL. Additionally, I form a "high-minus-low IVOL" zero-investment HV−LV IVOL portfolio with 666 monthly returns. The IVOL findings of Ang et al. (2006) are closely replicated and extended in Table 1.

[Table 1 here]

2. Graphical Diagnostic

The method I use is graphical and extends a similar diagnostic in Welch and Goyal (2008). I regress the HV−LV IVOL portfolio returns on standard monthly risk factors and estimate $\hat{\alpha} + \hat{\varepsilon}$ by pulling the coefficient factor product onto the left-hand side of the equation, conditional on the model.

As an example, to calculate the cumulative residuals for the 1F CAPM, I first regress the HV−LV IVOL portfolio returns on the market excess returns. I then subtract the coefficient factor product from the HV−LV IVOL portfolio returns, resulting in

$$R_{HV-LV,m,w} - \hat{\beta}_{MKT,m}\left(R_{MKT,m} - r_{f,m}\right) = \hat{\alpha}_m + \hat{\varepsilon}_m, \tag{2}$$

where $R_{HV-LV,m,w}$ is the HV−LV IVOL portfolio return in month m using weighting scheme w and $\hat{\beta}_{MKT,m}$ is the loading on the excess market return using prevailing data up to month m. I finally calculate the cumulative summation of $\hat{\alpha}_m + \hat{\varepsilon}_m$, i.e., for any month m, the quantity is $\sum_{n=1}^{m}(\hat{\alpha}_n + \hat{\varepsilon}_n)$, where $n = 1$ denotes the first observation of the sample.

The in-sample (IS hereafter) plots are created using the entire 666 HV−LV IVOL portfolio returns for the regression on the factors. The out-of-sample (OOS hereafter) plots differ in that they use only historically available data up to the time at which the forecast is made. As a result, the coefficients are updated every month. Since it is important to have a sufficient amount of initial data to have a reliable estimate of the coefficients, I similarly follow a time period specification from Welch and Goyal (2008) where the OOS forecasts begin after 20 observations are available. Therefore, the cumulative residual for any month m becomes $\sum_{n=20}^{m}(\hat{\alpha}_n + \hat{\varepsilon}_n)$.

The entire process is repeated for the other models by regressing the HV−LV IVOL portfolio returns on the factors of the Fama and French (1993) three-factor model, Fama and French (2015) five-factor model, and Fama and French (2018) six-factor model. The results are then plotted using both the value-weighted and equal-weighted HV−LV IVOL portfolio returns.[1]

The plots allow the reader to assess the effects of different starting and ending months (i.e., sample choice). The reader can visually shift the plot up or down according to the sample period of interest by moving the zero point axis to the start of the new period. The plots are

[1] In untabulated results, regressions of the value-weighted HV−LV IVOL portfolio return on the factors over the entire period from 1963 to 2018 show statistically significant abnormal returns for all models. The 5F alpha is −57 bps per month with a Newey and West (1987) adjusted T-statistic of −3.70. The 6F alpha is −39 bps per month with a Newey and West (1987) adjusted T-statistic of −2.39. The 1F and 3F alpha estimates are reported in Table 1.

normalized, i.e., they have vertically shifted the IS residuals so that the IS line begins at zero on the date of the first OOS. All figures are plotted using the same scale.

3. Results: The Empirical Performance of the IVOL Anomaly

Figure 1 presents the cumulative residuals from the value-weighted HV−LV IVOL portfolio for the IS and OOS. 1F and 3F refer to the CAPM and Fama-French (1993) three-factor model, respectively. The figure shows that the 1F-adjusted effect is strong from 1981 to 1998. Dividing the −522% difference in cumulative residuals by 18 years gives an alpha of approximately −29% per year. The 3F-adjusted effect is strong from 1965 to 2002 (with the exception of 1998 to 1999) and gives an alpha of approximately −18% per year. The plots show that the negative abnormal return drift continued after the 2000s, but at a slower rate. The "arrow periods" of Ang et al. (2006) and Bali and Cakici (2008) differ by only four years; therefore, they report similar findings regarding the value-weighted HV−LV IVOL portfolio.[2]

[Figure 1 here]

Figure 2 shows the plots for the equal-weighted HV−LV IVOL portfolio. Comparing this figure to Figure 1 suggests that the 1F-adjusted effect and 3F-adjusted effect are similar but weaker. Bali and Cakici (2008) report an insignificant effect for the equal-weighted HV−LV IVOL portfolio and Figure 2 confirms their findings.

[2] In untabulated results, I find that my estimates are also similar to those presented in Bali and Cakici (2008). For the sample period of Bali and Cakici (2008), i.e., July 1963 to December 2004, they report a 3F alpha of −127 bps per month with a Newey and West (1987) adjusted T-statistic of −6.33, and I find a 3F alpha of −130 bps per month with a Newey and West (1987) adjusted T-statistic of −6.27.

[Figure 2 here]

In Figure 3, my paper investigates the use of 5F and 6F benchmark return models. 5F and 6F refer to the five-factor model of Fama-French (2015) and the six-factor model of Fama-French (2018), respectively. Compared to Figure 1, it is even more evident in Figure 3 that since about 1998, there has been no 5F-adjusted and 6F-adjusted effect for the value-weighted HV−LV IVOL portfolio. Moreover, even the drift disappears.

[Figure 3 here]

Figure 4 shows the analogous plots for the equal-weighted HV−LV IVOL portfolio, again using 5F and 6F benchmark return models. The plots show that when returns are benchmarked to the 5F or 6F models, there has been no IVOL effect for the equal-weighted HV−LV IVOL portfolio.

One thing noticeable from Figures 3 and 4 is that, compared to Figures 1 and 2, the cumulative residuals are attenuated when using the 5F and 6F benchmark return models. However, the results only differ quantitatively, not qualitatively.

[Figure 4 here]

4. Conclusion

In sum, my paper finds that

1. From 1980 to about 2002 (with the exception of 1998 to 1999), the value-weighted IVOL portfolio had an alpha of about −24% per year.
2. Since 2002, the value-weighted effect has been muted.
3. The equal-weighted IVOL portfolio has mirrored the trend of the value-weighted portfolio, but had an alpha only one-third that of the value-weighted IVOL portfolio. Ergo, it was never significant.
4. The choice of the benchmark model is relatively unimportant.

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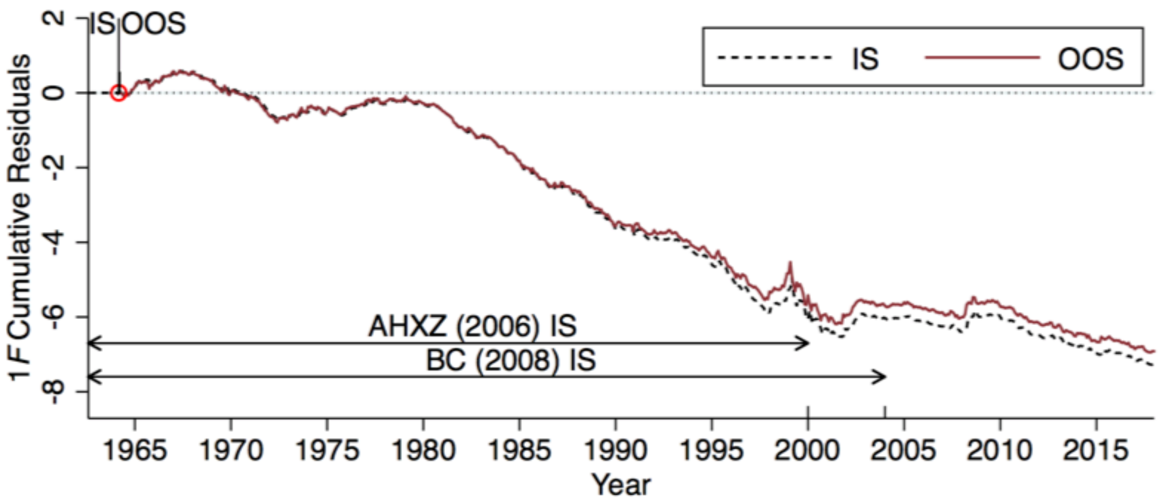
Table 1: Replication and Extension of Panel B of Table VI in Ang et al. (2006)

Description: This table shows key results from Panel B of Table VI in Ang et al. (2006). My replication for the same sample period, i.e., 1963 to 2000, as well as results for two different sample periods, 2001 to 2018 and 1963 to 2018, are reported. The row "HV−LV" refers to the difference in monthly returns between the portfolio of stocks with the highest IVOL and the portfolio of stocks with the lowest IVOL. The Alpha rows report alpha with respect to the CAPM (1F) or Fama and French (1993) three-factor model (3F) for the HV−LV IVOL portfolio. Robust Newey and West (1987) T-statistics are reported in square brackets.

Interpretation: The IVOL findings of Ang et al. (2006) are closely replicated and extended. For all sample periods, the raw average returns and alphas are lower for the portfolio of stocks with the highest IVOL than the portfolio of stocks with the lowest IVOL.

	Original	Replication	Update	
	July 1963 to Dec. 2000	July 1963 to Dec. 2000	Jan. 2001 to Dec. 2018	July 1963 to Dec. 2018
HV−LV	−1.06	−1.03	−0.19	−0.76
	[−3.10]	[−3.10]	[−0.36]	[−2.69]
1F Alpha	−1.38	−1.35	−0.70	−1.14
	[−4.56]	[−4.46]	[−2.01]	[−4.89]
3F Alpha	−1.31	−1.32	−0.82	−1.18
	[-7.00]	[−6.70]	[−2.72]	[−6.93]

Panel A: 1F (CAPM) Model



Panel B: 3F (Fama-French) Model

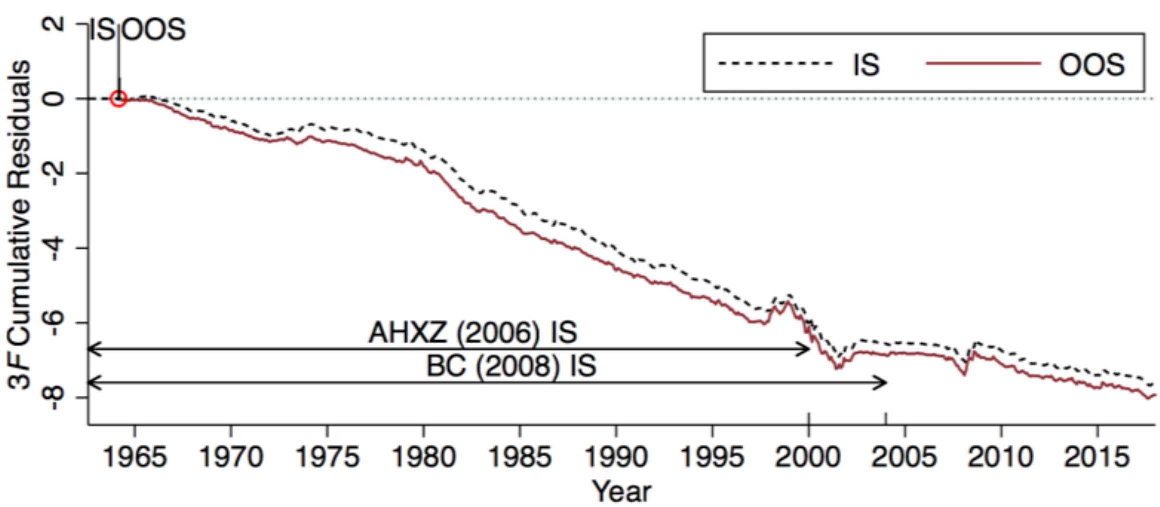
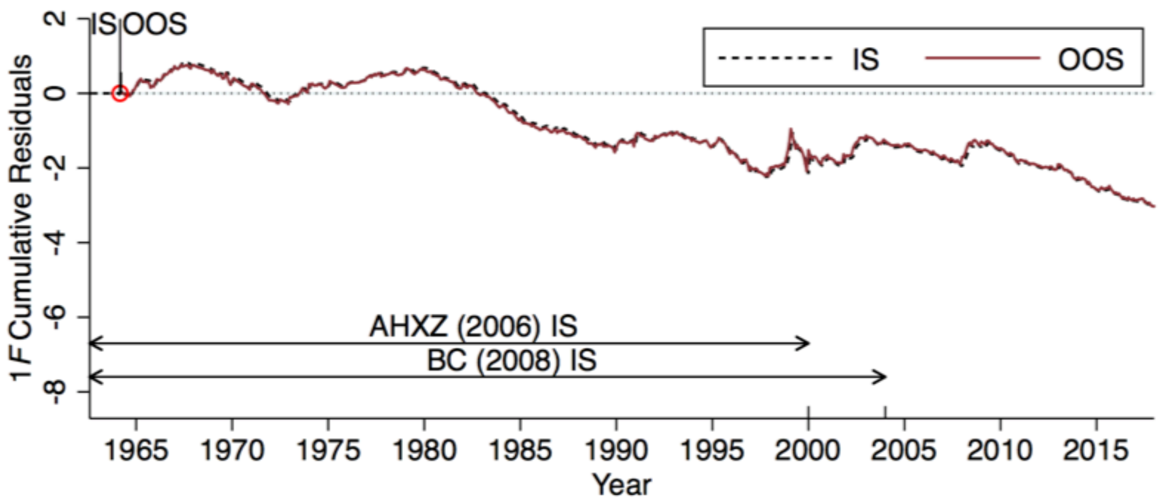


Figure 1: Cumulative Residuals for the Value-Weighted HV−LV IVOL Portfolio

Description: Panels A and B plot the cumulative residuals from the value-weighted HV−LV IVOL portfolio returns for the IS and OOS. It subtracts the coefficient factor product conditional on the model and estimates $\hat{\alpha} + \hat{\varepsilon}$. The OOS plots differ from the IS plots in that they use only historically available data up to the time at which the forecast is made. The cumulative residual for any month m is $\sum_{n=20}^{m}(\hat{\alpha}_n + \hat{\varepsilon}_n)$, since OOS forecasts begin after 20 observations. The plots have also vertically shifted the IS residuals so that the IS line begins at zero on the date of the first OOS. 1F and 3F refer to the CAPM and Fama-French (1993) three-factor model, respectively. These plots allow the assessment of the effects of different starting and ending months.

Interpretation: When 1F-adjusted, the IVOL effect is strong from 1981 to 1998. When 3F-adjusted, the IVOL effect is strong from 1965 to 2002 (with the exception of 1998 to 1999). The negative abnormal return drift continued after the 2000s, but at a much lower rate.

Panel A: 1F (CAPM) Model



Panel B: 3F (Fama-French) Model

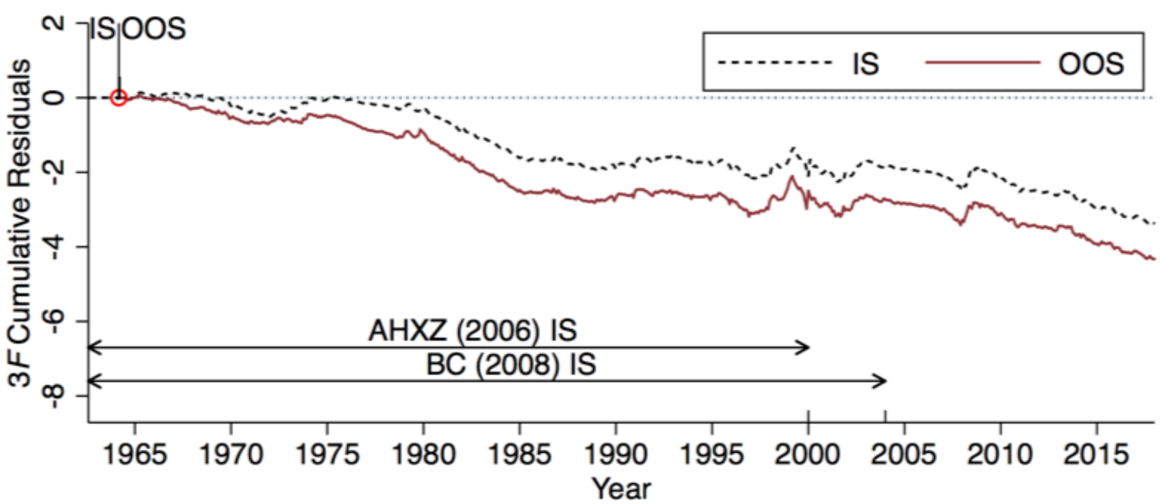
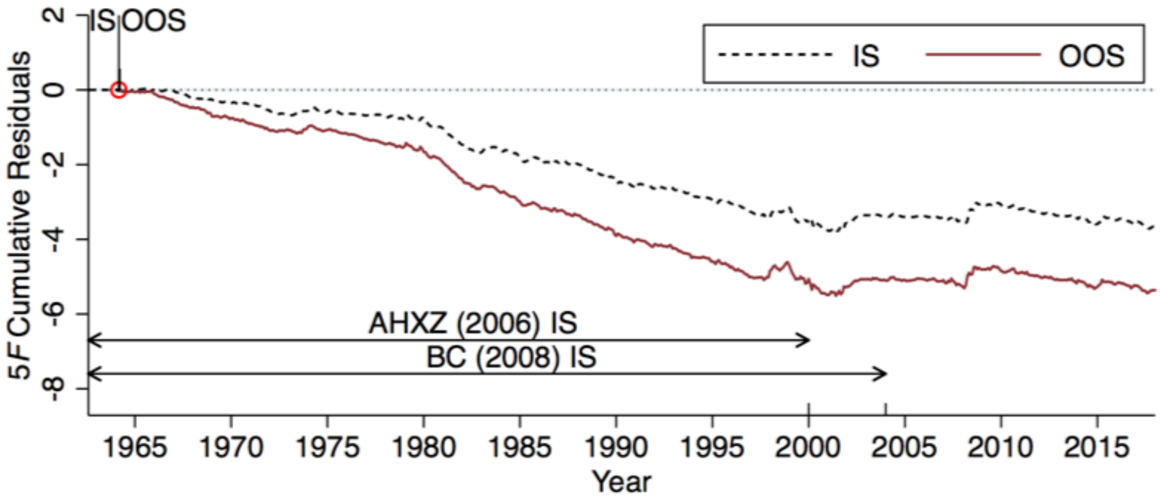


Figure 2: Cumulative Residuals for the Equal-Weighted HV−LV IVOL Portfolio

Description: See Figure 1, except the portfolio is equal-weighted. The same scale is used.

Interpretation: Compared to Figure 1, the 1F-adjusted effect and 3F-adjusted effect are similar but weaker. Not shown, they never reach statistical significance.

Panel A: 5F (Fama-French) Model



Panel B: 6F (Fama-French) Model

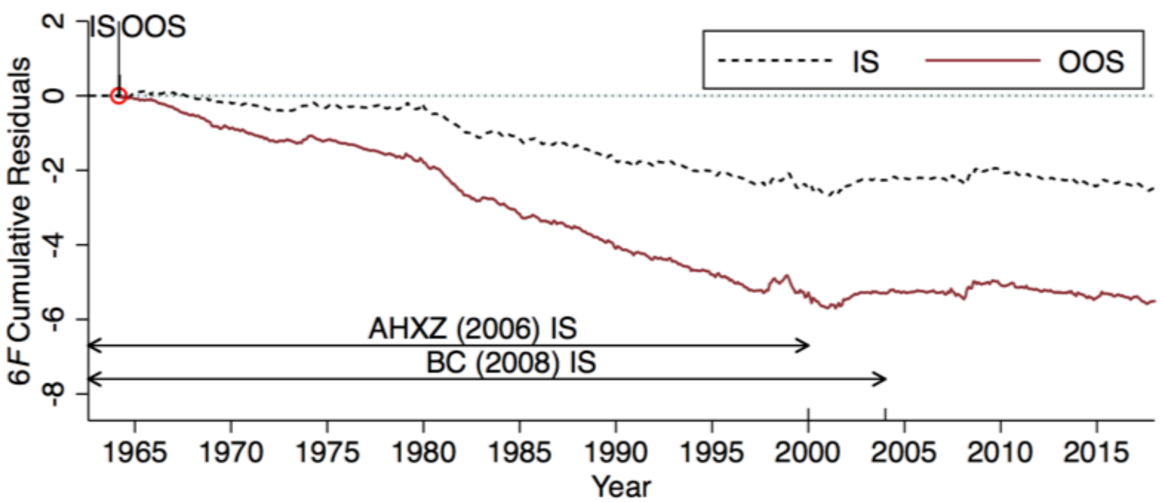
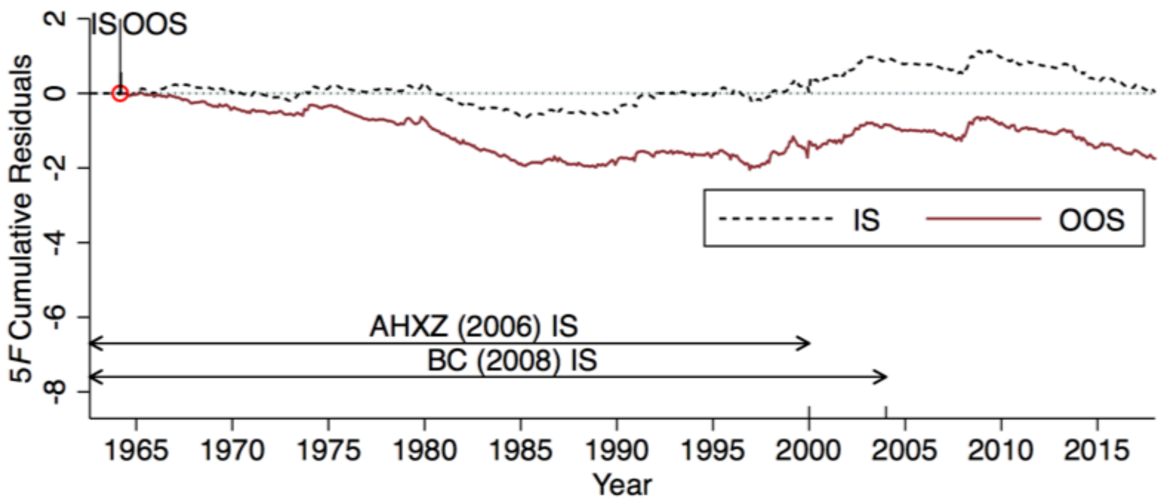


Figure 3: Cumulative Residuals for the Value-Weighted HV−LV IVOL Portfolio Using 5F and 6F Benchmark Return Models

Description: Like Figure 1, except the value-weighted IVOL portfolio is using the 5F and 6F benchmark return models. The same scale is used. 5F and 6F refer to the Fama-French (2015) five-factor and Fama-French (2018) six-factor model, respectively.

Interpretation: Using the 5F and 6F benchmark return models for the value-weighted HV−LV IVOL portfolio, the results are similar. Even clearer than in Figure 1, there has been no 5F-adjusted and 6F-adjusted effect since about 1998. Even the drift disappears.

Panel A: 5F (Fama-French) Model



Panel B: 6F (Fama-French) Model

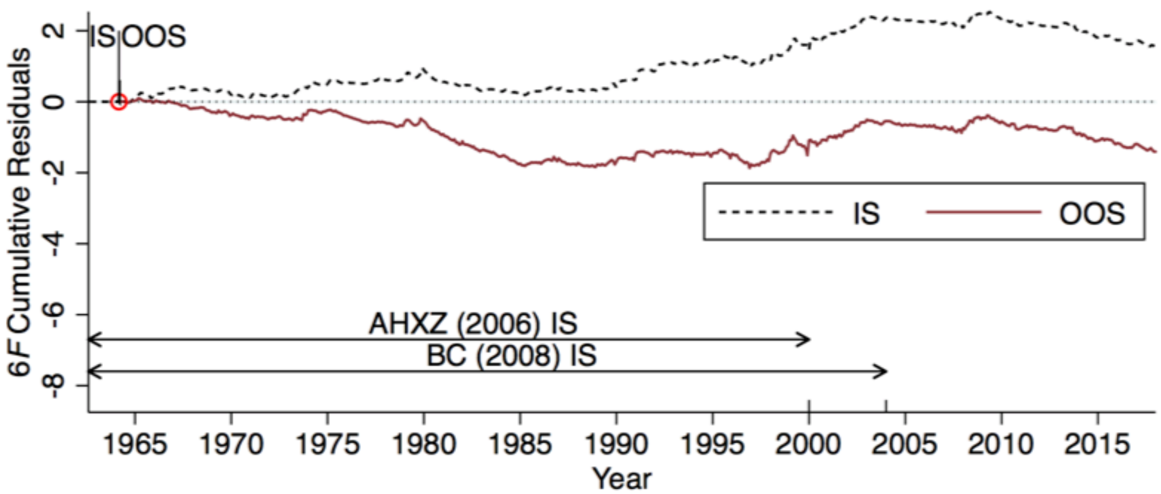


Figure 4: Cumulative Residuals for the Equal-Weighted HV−LV IVOL Portfolio Using 5F and 6F Benchmark Return Models

Description: See Figure 3, except the IVOL portfolio is equal-weighted. The same scale is used.

Interpretation: There never was an IVOL effect for the equal-weighted HV−LV IVOL portfolio when returns were benchmarked to the 5F or 6F models.