High Aversion to Stochastic Time Preference Shocks and Counterfactual Long-Run Risk in the Albuquerque et al. Valuation Risk Model*

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Does valuation risk induced by stochastic time preferences explain the equity premium puzzle as proposed by Albuquerque et al. (2016)? This explanation of the equity premium has several challenges. First, the valuation risk model implies extreme preference for early resolution of uncertainty and extreme aversion to valuation risk (which becomes infinite as elasticity of intertemporal substitution approaches one). Second, the model has a significant long-run risk component that counterfactually implies that consumption and dividend growth are highly persistent and predictable. Finally, I find no evidence that equity prices predict future risk-free rates as predicted by the baseline valuation risk model.

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Does valuation risk explain the equity premium and volatility of stock returns? The primary challenge faced by consumption-based asset pricing models is explaining the level and volatility of asset returns given the relative stability of consumption growth and its weak correlation with returns. This challenge is the essence of the Mehra and Prescott (1985) equity premium puzzle and the correlation puzzle discussed by an extensive literature including Campbell and Cochrane (1999) and Cochrane (2001).

Albuquerque, Eichenbaum, Luo, and Rebelo (2016) propose that valuation risk is the key to resolving these puzzles. In the valuation risk model, stochastic time preferences play a central role in generating stock price volatility. As agents become more impatient, discount rates rise and stock prices fall. With Epstein-Zin preferences, marginal utility varies with time preferences, inducing valuation risk. Stocks are risky because they perform poorly when investors become more impatient. Albuquerque et al. specify and estimate a structural model with stochastic time preferences. The model produces a large equity premium and volatile stock prices with consumption and dividend processes that are generally consistent with the data, and it accomplishes this with relatively low risk aversion of 1.5 to 2.4. Maurer (2012) proposes a similar model of stochastic time preferences generating a large equity premium. Schorfheide, Song, and Yaron (2018) and Creal and Wu (2020) model stochastic time preferences using Albuquerque et al.'s modified Epstein-Zin preferences and related processes for time preference variation.

In this paper, I assess the valuation risk model both theoretically and empirically. I start by considering the model's utility function with general consumption and time preference processes to derive general pricing results and develop intuition for how valuation risk affects asset prices. The resulting pricing equation indicates that stochastic time preferences create priced valuation risk relative to standard consumption models when elasticity of intertemporal substitution (EIS) differs from the inverse of the coefficient of relative risk aversion (RRA), which is what Albuquerque et al. (2016) find. The magnitude of the valuation risk premium is proportional to $\frac{RRA \times EIS - 1}{1 - EIS}$, indicating that valuation risk is governed by a combination of both RRA and EIS. Assessing the reasonableness of the model's RRA and EIS parameters in isolation misses the fact that seemingly reasonably parameters can imply extreme aversion to valuation risk. In particular, the model's valuation risk premium is infinite in the limit as EIS approaches one.[1] De Groot, Richter, and Throckmorton (2020) propose an alternative valuation risk model that does not have an infinite risk premium as EIS approches one and find that fixing this problem resurfaces the asset pricing puzzles that valuation risk was designed to solve.

I next assess the specific preferences implied by the valuation risk model by asking how much the model implies an agent would be willing to pay to avoid valuation risk. The answer is that in the benchmark model, agents would be willing to give up 90% of current and future consumption to avoid valuation risk by holding time preferences fixed, and in the extended valuation risk model, agents would be willing to give up 55% of current and future consumption to avoid valuation risk. These risk premia seem large and difficult to rationalize. At a minimum, they highlight that evaluating the model's preferences based solely on implied relative risk aversion and elasticity of intertemporal substitution is insufficient. I also follow Epstein, Farhi, and Strzalecki (2014) and evaluate the preference for early resolution of uncertainty implied by the valuation risk model by asking what fraction of current and future consumption agents would be willing to give up to resolve uncertainty immediately instead of gradually. The resulting timing premia of 82% for the baseline model and 55% for the extended model are also large and difficult to rationalize, particularly given that Epstein, Farhi, and Strzalecki question the plausibility of much smaller timing premia of 24 to 31% implied by Bansal and Yaron's (2004) long-run risk model.

The paper's empirical analysis starts by assessing what drives the equity premium in the valuation risk model. While valuation risk plays an important role, long-run risk associated with persistent consumption and dividend growth explains over half of the equity premium in the extended Albuquerque et al. (2016) model. As a result, Albuquerque et al. and Bansal

[1] Uhlig (2014) makes a similar argument in a conference discussion of Albuquerque et al. (2016) that was contemporaneous with earlier versions of this paper.

and Yaron (2004) have similar implications for consumption and dividend growth persistence and predictability. I follow Beeler and Campbell's (2012) assessment of Bansal and Yaron's long-run risk model to evaluate these predictions. Consumption and dividend growth are more persistent and predictable by the price-dividend ratio in the valuation risk model than they are in the long-run risk model, which is inconsistent with the data.

Finally, the price-dividend ratio does not predict future risk-free rates in the data. In contrast, the benchmark model counterfactually generates strong risk-free rate predictability. This predictability is the essence of valuation risk. Investor impatience increases discount rates, causing prices to fall. While the extended model is more consistent with the data, the empirical lack of a relation between equity prices and future risk-free rates highlights that there is little direct support for the valuation risk model in the data.

The challenges to valuation risk described in this paper highlight important limitations to stochastic time preferences within a representative agent model. Heterogeneous agent models such as those proposed by Bhamra and Uppal (2014) and Garleanu and Panageas (2015) avoid these issues by generating discount rate variation through endogenous changes to wealth over time instead of through preference shocks. While the valuation risk model is a step forward for understanding how stochastic time preferences can affect asset prices, the preference assessments and empirical evidence in this paper cast doubt on valuation risk's ability to resolve asset pricing puzzles. More generally, the paper highlights complications of adding stochastic preferences to standard utility functions.[2] Preference risk is fundamentally different from consumption risk and likely requires more flexible preference models.

1 Theory

Following Albuquerque et al. (2016), I consider a representative agent with constant elasticity of substitution Kreps and Porteus (1978) preferences characterized by a recursive

[2]Stochastic preferences can also have problematic implications for real macroeconomic activity as discussed by de Groot, Richter, and Throckmorton (2018).

utility function similar to Weil (1989) and Epstein and Zin (1991). The only change from standard Epstein-Zin utility is the addition of stochastic time preferences. I start by considering general consumption and time-preference processes and then discuss the specific model proposed by Albuquerque et al.. The main results are presented and discussed below. Derivations and additional details are in the Internet Appendix.

The representative agent's preferences are summarized by continuation utility U_t, which satisfies

$$U_t = \left[\lambda_t C_t^{1-1/\psi} + \delta \left(\mathrm{E}_t \left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{1/(1-1/\psi)} \tag{1}$$

where C_t is consumption at time t, $\delta < 1$ is a positive scalar capturing time discounting, ψ is elasticity of intertemporal substitution, and γ is the coefficient of relative risk aversion. The function is defined for $\psi \neq 1$ and $\gamma \neq 1$. Epstein and Zin (1989) prove the existence of recursive utility functions of this form without the λ_t term for a specified domain of consumption programs. Equation (1) represents standard Epstein-Zin preferences except that time preferences are allowed to vary over time instead of being constant. Time preferences are affected by $\frac{\lambda_{t+1}}{\lambda_t}$, which is assumed to be known at time t. Because λ_t has a mean growth rate of zero in the models being considered, the consumption program domain for equation (1) is essentially the same as it is for standard Epstein-Zin preferences.[3]

Using standard techniques for working with Epstein-Zin preferences, Albuquerque et al. (2016) show that equation (1) implies a log stochastic discount factor of

$$m_{t+1} = \theta \log(\delta) + \theta \Lambda_{t+1} - \frac{\theta}{\psi}\Delta c_{t+1} + (\theta - 1) r_{w,t+1} \tag{2}$$

where $\Lambda_{t+1} = \log\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$ and $\theta = \frac{1-\gamma}{1-1/\psi}$. Lower case letters signify logs. Log consumption growth from period t to period $t+1$ is Δc_{t+1}. The log return on the overall wealth portfolio, which is the claim to aggregate consumption, is $r_{w,t+1}$. This stochastic discount factor is

[3]See Epstein and Zin (1989) for a formal description of the consumption program domain. The main restriction is that long-run consumtion growth must be less than $(\frac{1}{\delta})^{1/(1-1/\psi)}$. In the modified Epstein-Zin preferences described by equation (1), this restriction applies to $\lambda_t^{1/(1-1/\psi)} C_t$.

standard for Epstein-Zin preferences except that time discounting (δ) is augmented by $\frac{\lambda_{t+1}}{\lambda_t}$.

1.1 General model

First, consider an endowment economy with general processes for consumption and time preferences to derive general pricing results and develop intuition about how valuation risk affects prices. Innovations to current and expected future consumption growth and time preferences are jointly lognormal and homoscedastic. Specifically,

$$\mathrm{E}_t\left[\Delta c_{t+a}\right] = \mathrm{E}_{t-1}\left[\Delta c_{t+a}\right] + \varepsilon^c_{a,t} \tag{3}$$

and

$$\mathrm{E}_t\left[\Lambda_{t+1+b}\right] = \mathrm{E}_{t-1}\left[\Lambda_{t+1+b}\right] + \varepsilon^\lambda_{b,t} \tag{4}$$

with $\left\{\varepsilon^c_{a,t}\right\}_{a>0}, \left\{\varepsilon^\lambda_{b,t}\right\}_{b>0}$ distributed jointly normally with constant variance.[4] This assumption implies that excess returns on the wealth portfolio are also lognormal and homoscedastic. For simplicity, I assume that all other excess returns are lognormal as well. Lognormality and homoscedasticity simplify the model and ensure that risk premia are constant over time, focusing attention on consumption growth and time preference shocks. In their benchmark model, Albuquerque et al. (2016) specify a more restrictive stochastic process for time preferences and assume that expected consumption growth is constant over time. Albuquerque et al.'s extended model adds variance shocks and specifies a more general stochastic process for time preferences and consumption growth. Similarly, Bansal and Yaron's (2004) case 1 model specifies consumption growth shocks that nest within the structure specified by equation (3), and their case 2 model adds variance shocks.

The stochastic discount factor of equation (2) can be used to price all assets. In particular,

[4]Note that Λ_{t+1} is known one period in advance so time t shocks to Λ expectations start with Λ_{t+1}.

it implies a real risk-free rate of

$$r_{f,t+1} = -\log(\delta) - \Lambda_{t+1} + \frac{1}{\psi}\mathrm{E}_t[\Delta c_{t+1}] - \frac{1-\theta}{2}\sigma_w^2 - \frac{\theta}{2\psi^2}\sigma_c^2 \tag{5}$$

and risk premia of

$$\mathrm{E}_t[r_{i,t+1}] - r_{f,t+1} + \frac{1}{2}\sigma_i^2 = \frac{\theta}{\psi}\sigma_{ic} + (1-\theta)\sigma_{iw} \tag{6}$$

where σ_w^2 is the variance of excess returns to the wealth portfolio, $\sigma_c^2 = var_t\left(\varepsilon_{0,t+1}^c\right)$ is the conditional variance of consumption growth, σ_{ic} is covariance of asset i's return with current consumption shocks, and σ_{iw} is covariance of asset i's return with wealth portfolio returns. Subtracting $\frac{1}{2}\sigma_i^2$ is a Jensen's inequality correction for expected log returns using the variance of asset i's return. From equations (5) and (6), it is clear that the real risk-free interest rate changes over time in response to time preferences (Λ_{t+1}) and expected consumption growth ($\mathrm{E}_t[\Delta c_{t+1}]$) and that risk premia are constant over time.

1.1.1 Extended consumption CAPM

The representative agent's budget constraint is $W_{t+1} = R_{w,t+1}(W_t - C_t)$, where W_t is wealth and $R_{w,t+1}$ is the gross return to the wealth portfolio. Following Campbell (1993, 2018), the budget constraint can be log-linearized to yield

$$r_{w,t+1} - \mathrm{E}_t[r_{w,t+1}] = (\mathrm{E}_{t+1} - \mathrm{E}_t)\sum_{j=0}^{\infty}\rho^j \Delta c_{t+1+j} - (\mathrm{E}_{t+1} - \mathrm{E}_t)\sum_{j=1}^{\infty}\rho^j r_{w,t+1+j} \tag{7}$$

where ρ is a log-linearization constant.[5] Because risk premia are constant over time, shocks to expected future returns, $News_{h,t+1} \equiv (\mathrm{E}_{t+1} - \mathrm{E}_t)\sum_{j=1}^{\infty}\rho^j r_{w,t+1+j}$, depend solely on changes to expected interest rates, which change over time in response to time preferences and expected consumption growth as described by equation (5).[6]

Using the budget constraint specified by equation (7) and the risk-free rate decomposition

[5] Specifically, $\rho = 1 - \exp\left(\overline{c-w}\right)$ where $\overline{c-w}$ is the average log consumption-wealth ratio.
[6] The h subscript follows the notation of Campbell (1993) to indicate hedging of future interest rates.

of equation (5), wealth portfolio returns can be substituted out to express the stochastic discount factor and risk premium equation as an extended consumption capital asset pricing model (CCAPM).The resulting log stochastic discount factor is

$$\begin{aligned} m_{t+1} &= \theta \log(\delta) + \theta \Lambda_{t+1} - \frac{\theta}{\psi} \mathrm{E}_t \Delta c_{t+1} - \gamma \left(\Delta c_{t+1} - \mathrm{E}_t \Delta c_{t+1} \right) \\ &\quad + \left(\frac{1}{\psi} - \gamma \right) \left(\mathrm{E}_{t+1} - \mathrm{E}_t \right) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \\ &\quad + \frac{1-\gamma\psi}{\psi - 1} \left(\mathrm{E}_{t+1} - \mathrm{E}_t \right) \sum_{j=1}^{\infty} \rho^j \Lambda_{t+1+j}, \end{aligned} \tag{8}$$

and the resulting pricing equation is

$$\mathrm{E}_t \left[r_{i,t+1} \right] - r_{f,t+1} + \frac{1}{2} \sigma_i^2 = \gamma \sigma_{ic} + (\gamma\psi - 1) \sigma_{ih(c)} - \frac{\gamma\psi - 1}{\psi - 1} \sigma_{ih(\lambda)}. \tag{9}$$

where $\sigma_{ih(c)} \equiv cov_t \left(r_{i,t+1}, (\mathrm{E}_{t+1} - \mathrm{E}_t) \sum_{j=1}^{\infty} \rho^j \frac{1}{\psi} \Delta c_{t+1+j} \right)$ is covariance with shocks to expected future interest rates due to changing consumption growth expectations, and $\sigma_{ih(\lambda)} \equiv cov_t \left(r_{i,t+1}, (\mathrm{E}_{t+1} - \mathrm{E}_t) \sum_{j=1}^{\infty} \rho^j \Lambda_{t+1+j} \right)$ is covariance with shocks to expected future interest rates due to changing expected time preferences. Together, they add up to covariance with overall interest rate news, $\sigma_{ih} \equiv cov_t \left(r_{i,t+1}, (\mathrm{E}_{t+1} - \mathrm{E}_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j} \right) = \sigma_{ih(c)} + \sigma_{ih(\lambda)}$. Derivation details and an alternative intertemporal CAPM representation of the pricing equation are in the Internet Appendix.

Equation (9) is an extended version of the standard consumption CAPM pricing equation. As in other CCAPM models, consumption risk (σ_{ic}) is priced by relative risk aversion (γ). Consistent with Bansal and Yaron (2004), the standard CCAPM holds under power utility ($\gamma = 1/\psi$), and covariance with shocks to expected future consumption growth ($\sigma_{ih(c)}$) is only priced if $\gamma \neq 1/\psi$.[7] Covariance with shocks to expected future time preferences ($\sigma_{ih(\lambda)}$) is also priced only if $\gamma \neq 1/\psi$. Yet, the two types of interest rate news covariance are priced

[7] Bansal and Yaron (2004) express their version of equation (9) in terms of covariance with future consumption growth instead of covariance with risk-free rate news. This is a different way of describing the same relation.

differently. Whereas $\sigma_{ih(c)}$ is priced by $\gamma\psi - 1$, $\sigma_{ih(\lambda)}$ is priced by $-\frac{\gamma\psi-1}{\psi-1}$. When $\psi > 1$, the prices have opposite signs, and if ψ is close to 1, time-preference risk is amplified relative to consumption growth risk.

1.1.2 Augmented consumption

Another way to derive the extended CCAPM pricing equation is to change notation and consider preferences with respect to augmented consumption, defined as $\widetilde{C}_t \equiv \lambda_t^{1/(1-1/\psi)} C_t$. With this notation change, equation (1) is transformed into standard Epstein-Zin preferences with respect to augmented consumption. Log augmented consumption growth, $\Delta\widetilde{c}_{t+1} = \Delta c_{t+1} + \frac{1}{1-1/\psi}\Lambda_{t+1}$ comes from both consumption growth and time preferences. As Albuquerque et al. (2016) note, time preferences and consumption growth operate in similar ways. Dew-Becker and Giglio (2016) show that under typical calibrations, Epstein-Zin preferences imply large risk prices for long-run, low frequency consumption growth shocks. The same thing is true for long-run shocks to $\frac{1}{1-1/\psi}\Lambda_{t+1}$ under the modified Epstein-Zin preferences described by equation (1). Standard pricing equations hold with respect to augmented consumption, and equation (9) can be obtained by a change of variables transformation from augmented consumption to consumption.

1.1.3 Valuation risk as ψ approaches 1

Equation (1) is not defined when $\psi = 1$, and the valuation risk premium ($-\frac{\gamma\psi-1}{\psi-1}\sigma_{ih(\lambda)}$ in Equation (9)) becomes infinite as ψ approaches 1. As can be seen in equation (8), the log stochastic discount factor's variance becomes infinite as ψ approaches 1. The only way to avoid this result is for variance of time preference shocks to approach zero as ψ approaches 1.

Consider alternative preferences described by utility V_t satisfying recursion

$$V_t = \left[(\lambda_t^* C_t)^{1-1/\psi} + \delta \left(\mathrm{E}_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1-1/\psi}{1-\gamma}} \right]^{1/(1-1/\psi)} \tag{10}$$

when $\psi \neq 1$, and

$$\log\left(V_t\right) = \log\left(\lambda_t^* C_t\right) + \delta \log\left(\mathrm{E}_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{1-\gamma}} \tag{11}$$

when $\psi = 1$. Modified time preference λ_t^* is a multiplier on consumption, whereas λ_t in equation (1) is a multiplier on the flow utility from consumption, $C_t^{1-1/\psi}$. Utility function V_t represents standard Epstein-Zin preferences with respect to $\lambda_t^* C_t$, and V_t is equivalent to U_t in equation (1) when $\psi \neq 1$ and $\lambda_t^* = \lambda_t^{1/(1-1/\psi)}$.

Substituting λ_t^* for λ_t in equations (8) and (5) yields a log stochastic discount factor of

$$\begin{aligned} m_{t+1} \quad = \quad & \theta \log\left(\delta\right) + \left(1-\gamma\right) \Lambda_{t+1}^* - \frac{\theta}{\psi} \mathrm{E}_t \Delta c_{t+1} - \gamma\left(\Delta c_{t+1} - \mathrm{E}_t \Delta c_{t+1}\right) \\ & + \left(\frac{1}{\psi} - \gamma\right)\left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \\ & + \left(\frac{1}{\psi} - \gamma\right)\left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{\infty} \rho^j \Lambda_{t+1+j}^* \end{aligned} \tag{12}$$

and a risk-free rate of

$$r_{f,t+1} = -\log\left(\delta\right) - \left(1 - \frac{1}{\psi}\right) \Lambda_{t+1}^* + \frac{1}{\psi} \mathrm{E}_t\left[\Delta c_{t+1}\right] - \frac{1-\theta}{2} \sigma_w^2 - \frac{\theta}{2\psi^2} \sigma_c^2, \tag{13}$$

where $\Lambda_{t+1}^* = \log\left(\frac{\lambda_{t+1}^*}{\lambda_t^*}\right)$. If $\left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{\infty} \rho^j \Lambda_{t+1+j}^*$ has finite variance, the log stochastic discount factor has finite variance, and risk premia are finite, even when $\psi = 1$.

The preferences described by equations (10) and (11) are well-defined and generate finite risk premia when $\psi = 1$. While this is a technical fix to the model's infinite valuation risk premia when $\psi = 1$, the fix works by eliminating valuation risk as ψ approaches 1. When $\psi = 1$, the risk-free rate in equation (13) is insensitive to Λ_{t+1}^*. Similarly, the rate of time preference implied by equations (10) and (11), $\delta\left(\frac{\lambda_{t+1}^*}{\lambda_t^*}\right)^{1-1/\psi}$, is insensitive to λ_{t+1}^* when $\psi = 1$. Thus, when $\psi = 1$, shocks to λ_{t+1}^* are priced, but they are unobserved and have no impact on time preferences or valuations.

Under the original valuation risk model, valuation risk premia are infinite as ψ approaches

1. Under the alternative preferences specified by equations (10) and (11), valuation risk premia are finite, but the underlying valuation risk becomes infinitesimally small as ψ approaches 1. Finite risk premia for infinitesimal shocks are just as implausible as infinite risk premia for finite shocks. In addition to embedding implausible preferences, the valuation risk model also becomes difficult to test in the data as ψ approaches 1. When ψ is close to 1, agents are extremely averse to arbitrarily small future preference changes that have almost no impact on time preferences, the risk-free rate, or asset valuations. If the unobserved shocks have no little impact observed prices, it is difficult to test the model in the data.

1.1.4 Valuation risk aversion

The extended CCAPM pricing equation (9) highlights that valuation risk becomes increasingly important as ψ approaches one. In the limit as ψ approaches one, valuation risk premia become infinite, and the model becomes untestable. This is problematic because ψ is the model's elasticity of intertemporal substitution (EIS), which is frequently estimated and calibrated as being close to one. For example, Hansen and Sargent (2008) develop a model with an EIS of one, Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012) calibrate EIS as 1.5, and Albuquerque et al. (2016) estimate EIS to be 1.5 to 2.2.[8] Infinite valuation risk premia are implausible. The fact that the valuation risk model generates infinite risk premia with an EIS that is considered reasonable by much of the literature highlights the multiple roles that γ and ψ play in the preferences expressed by equation (1). In addition to capturing risk aversion and EIS, equation (1) also embeds preferences related to valuation risk and resolution of uncertainty. The plausibility of the modeled preferences depends not just on whether relative risk aversion and EIS are reasonable in isolation but also on these additional preferences. Seemingly reasonable parameters can imply extreme valuation risk aversion, potentially creating large risk premia with small preference shocks that may be

[8] An extensive literature empirically estimating EIS has produced little consensus with results ranging from close to zero (e.g., Hall, 1988 and Campbell, 2003) to over one (e.g., Beaudry and van Wincoop, 1996 and Gruber, 2013).

difficult to detect in the data. Section 2 revisits valuation risk aversion in more detail and assesses the specific preferences implied by the Albuquerque et al. valuation risk model.

1.2 Calibrated model

Albuquerque et al. (2016) first propose a benchmark model of an endowment economy with the following process for consumption growth, dividend growth, and time preferences:

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + \sigma \varepsilon^c_{t+1} \\ \Delta d_{t+1} &= \mu_d + \pi_{dc}\sigma\varepsilon^c_{t+1} + \varphi\sigma\varepsilon^d_{t+1} \\ \Lambda_{t+1} &= \rho_\Lambda \Lambda_t + \sigma_\Lambda \varepsilon^\Lambda_t \\ & \quad \varepsilon^c_{t+1}, \varepsilon^d_{t+1}, \varepsilon^\Lambda_t \overset{iid}{\sim} \mathcal{N}(0,1). \end{aligned} \tag{14}$$

The log time preference ratio, Λ_{t+1}, is the only persistent state variable in the economy. Variability of time preference shocks is determined by σ_Λ, and ρ_Λ determines their persistence. At time t, ε^Λ_t and Λ_{t+1} are both known. The model is a special case of the general model discussed in Section 1.1 with constant expected consumption and dividend growth and shocks to current and expected time preferences determined by ε^Λ_t.

In their extended model, Albuquerque et al. (2016) consider an endowment economy with a more general process for consumption growth, dividend growth, and time preferences:

$$\begin{aligned} \Delta c_{t+1} &= \mu_c + \rho_c \Delta c_t + \alpha_c\left(\sigma^2_{t+1} - \sigma^2\right) + \pi_{c\Lambda}\varepsilon^\Lambda_{t+1} + \sigma_t \varepsilon^c_{t+1} \\ \Delta d_{t+1} &= \mu_d + \rho_d \Delta d_t + \alpha_d\left(\sigma^2_{t+1} - \sigma^2\right) + \pi_{d\Lambda}\varepsilon^\Lambda_{t+1} + \pi_{dc}\sigma_t\varepsilon^c_{t+1} + \varphi\sigma_t\varepsilon^d_{t+1} \\ \sigma^2_{t+1} &= \sigma^2 + \nu\left(\sigma^2_t - \sigma^2\right) + \sigma_\omega \omega_{t+1} \\ \Lambda_{t+1} &= x_t + \sigma_\eta \eta_{t+1} \\ x_{t+1} &= \rho_\Lambda x_t + \sigma_\Lambda \varepsilon^\Lambda_{t+1} \\ & \quad \varepsilon^c_{t+1}, \varepsilon^d_{t+1}, \varepsilon^\Lambda_{t+1}, \omega_{t+1}, \eta_{t+1} \overset{iid}{\sim} \mathcal{N}(0,1). \end{aligned} \tag{15}$$

Here, σ_{t+1}^2 is time-varying volatility, which is centered at and slowly reverts to σ^2. The current log time preference ratio, Λ_{t+1}, is impacted both by transitory shocks, η_{t+1}, and by a persistently varying component, x_t. At time t, η_{t+1} is known so that Λ_{t+1} is known one period in advance.

Compared to the benchmark model, the extended model adds time-varying volatility, persistence in consumption and dividend growth, transitory time preference shocks, and dependence of consumption and dividend growth on time preference shocks. The extended model also includes persistent changes to expected consumption and dividend growth through the $\alpha_c\left(\sigma_{t+1}^2 - \sigma^2\right)$ and $\alpha_d\left(\sigma_{t+1}^2 - \sigma^2\right)$ terms. These persistent changes to expected growth rates have the effect of embedding long run risk within the model.[9] With $\sigma_\omega = 0$, the model is a special case of the more general model discussed in Section 1.1. With $\sigma_\omega \neq 0$, the extended model adds time-varying conditional variance.

Albuquerque et al. (2016) solve the model with log-linear approximations (see the Internet Appendix for details) and estimate the model using simulated method of moments and historical data on consumption, dividends, and returns. Table **1** reports Albuquerque et al.'s parameter estimates.[10] Of particular note, the benchmark and extended models both produce reasonably low relative risk aversion estimates (1.51 and 2.40, respectively). The benchmark model captures the basic elements of valuation risk. The extended model generates more empirically realistic consumption growth, dividend growth, and returns.

[Insert Table **1** Here]

[9]In the Bansal and Yaron (2004) long run risk model, dividend and consumption growth have a small persistent predictable component. Because expected consumption and dividend growth depend on volatility, stochastic volatility introduces a similar persistent predictable component to consumption and dividend growth in the extended valuation risk model.

[10]Parameters are the same as those reported by Albuquerque et al. (2016) except that I define π_{dc} and φ as multipliers of σ instead of as standard deviations in the benchmark model for consistency with the extended model. The parameters satisfy the restriction that long-run consumtion growth must be less than $(\frac{1}{\delta})^{1/(1-1/\psi)}$, which corresponds to annual consumption growth of 8.2% in the baseline model and 4.6% in the extended model.

2 Preference assessment

The central contribution of the valuation risk model is that the "model accounts for the equity premium and volatility of stock and bond returns, even though the estimated degree of agents' risk aversion is moderate (roughly 1.5)" (Albuquerque et al., 2016, p. 2863). The model's elasticity of intertemporal substitution (roughly 1.5 to 2.2) is also in a range typically considered reasonable. This contribution is analogous to the long-run risk model of Bansal and Yaron (2004) and is arguably more significant because "long-run risk models require a high degree of risk aversion to match the equity premium" (Albuquerque et al., 2016, p. 2883).[11] Fundamentally, this is a quantitative contribution. The model generates an empirically reasonable equity premium while matching other moments in the data with preference parameters that are seemingly reasonable based on introspection and experimental research.

With respect to preferences, the valuation risk model's estimated risk aversion and elasticity of intertemporal substitution are moderate. However, risk aversion and intertemporal substitution are not the only relevant preferences. As Epstein, Farhi, and Strzalecki (2014) emphasize in their evaluation of long-run risk models, Epstein-Zin utility also implies a preference for early or late resolution of uncertainty. The valuation risk model utilizes not just Epstein-Zin utility, but a modified version of Epstein-Zin utility that includes stochastic time preferences. Thus, while preferences specified by the valuation risk model utility function (equation (1)) are summarized by two parameters (γ and ψ), these parameters govern not just relative risk aversion and elasticity of intertemporal substitution, but also preferences for resolution of uncertainty and valuation risk. Evaluating the reasonableness of γ and ψ only for their relative risk aversion and elasticity of intertemporal substitution implications creates an incomplete picture of the overall plausibility of the preferences implied by equation (1).

To assess the overall preferences implied by the valuation risk model, I follow Epstein,

[11]The long run risk model of Bansal and Yaron (2004) requires risk aversion on the order of 7.5 to 10.

Farhi, and Strzalecki (2014) and ask, "How much would you pay to resolve long-run risk?" I.e., how much consumption would you give up to immediately resolve all future uncertainty? To assess aversion to time preference risk, I analogously ask how much agents would pay to eliminate time preference risk by holding λ_t constant for all future periods.

2.1 Timing premium

Epstein, Farhi, and Strzalecki (2014) propose the following thought experiment. Consider a given consumption process with uncertainty resolved over time and the same consumption process with all uncertainty resolved at time 1. Both options involve the same consumption process and same risk. The only difference is when the uncertainty is resolved. Epstein-Zin utility with $\psi > \frac{1}{\gamma}$ implies that one prefers early resolution. To quantify the strength of this preference, Epstein, Farhi, and Strzalecki propose considering a timing premium, π^*, defined as the maximum fraction of current and future consumption one would be willing to give up to resolve all uncertainty at time 1. Defining U_0 as the utility of the consumption process with gradual resolution of risk and U_0^* as the utility of the same consumption process with all risk resolved at time 1, the timing premium is

$$\pi^* = 1 - \frac{U_0}{U_0^*}. \tag{16}$$

As Albuquerque et al. (2016) discuss, the valuation risk model generates large, persistent movements in the stochastic discount factor, much like the long run-run risk model. As a result, the valuation risk model potentially embeds a similar timing premium. Calculating the timing premium implied by the valuation risk model's consumption process and preferences requires calculating U_0 and U_0^* using numerical methods. Note that U_t in equation (1) can be expressed as

$$U\left(C_t, \frac{C_t}{C_{t-1}}, \lambda_t, \lambda_{t+1}, x_t, \sigma_t^2\right) = C_{t-1}\lambda_t^{\frac{1}{1-1/\psi}} H\left(\Delta c_t, \Lambda_{t+1}, x_t, \sigma_t^2\right), \tag{17}$$

where $H : \mathbb{R}^4 \to \mathbb{R}$ is the solution to

$$H\left(\Delta c, \Lambda, x, \sigma^2\right) = \left\{\exp\left(\Delta c\right)^{1-1/\psi}\left[1 + \delta \exp\left(\Lambda\right) J\left(\Delta c, x, \sigma^2\right)^{\frac{1-1/\psi}{1-\gamma}}\right]\right\}^{\frac{1}{1-1/\psi}}, \tag{18}$$

and $J : \mathbb{R}^3 \to \mathbb{R}$ is

$$J\left(\Delta c, x, \sigma^2\right) \equiv \mathrm{E}_{\Delta c, x, \sigma^2}\left[H\left(\Delta c', \Lambda', x', \sigma^{2\prime}\right)^{1-\gamma}\right]. \tag{19}$$

Here, $\mathrm{E}_{\Delta c, x, \sigma^2}$ is the expectation conditional on Δc, x, and σ^2. This is the same basic approach taken by Epstein, Farhi, and Strzalecki (2014) with a little more algebra because the valuation risk model has a larger set of state variables. I then approximate $J\left(\Delta c, x, \sigma^2\right)$ on a discrete grid of Δc, x, and σ^2 using Monte Carlo simulation to approximate the expectation and iterating to find a fixed point for $J\left(\Delta c, x, \sigma^2\right)$. This approach achieves a reasonable level of precision with 5,000 iterations, approximating the expectation operator with 1,000 random simulations in the final iterations. Sensitivity analysis on the grid, number of iterations, and number of simulations indicates that the resulting estimate is a stable and accurate approximation of U_0.

To calculate the value of U_0^*, I follow Epstein, Farhi, and Strzalecki (2014) and run Monte Carlo simulations of U_1^*. Because all uncertainty is resolved at time 1, each simulation corresponds to a realized consumption and time preference path. To calculate U_1^*, I simulate paths of length $T = 5,000$ with continuation value U_0. I repeat this process 100,000 times and then compute U_0^* using equation (1).

Table **2** reports the resulting timing premium. In the benchmark model, π^* is 82%, and π^* is 55% for the extended model, which means the valuation risk model implies agents would be willing to give up over half of their current and future consumption to resolve future uncertainty immediately at time 1. Is this level of aversion to gradual resolution of uncertainty reasonable? Epstein, Farhi, and Strzalecki (2014) find significantly smaller timing premia of 20% to 30% for long-run risk models and argue that such preferences are difficult to rationalize. The analysis implies that agents in the valuation risk model would be willing to give up most of their lifetime consumption to change the timing of uncertainty resolution without any impact on the underlying consumption and time preference process or risk level. While theoretically possible, aversion to gradual resolution of uncertainty of this magnitude is difficult to rationalize. At a minimum, these timing premia suggest that the valuation risk model's preference estimates are not as moderate as looking at γ and ψ in isolation would suggest.

[Insert Table 2 Here]

2.2 Valuation risk premium

Next, consider an analogous question about valuation risk. How much would you pay to eliminate valuation risk? To answer this question, I calculate valuation risk premium, $\widehat{\pi}$, defined as the maximum fraction of current and future consumption one would be willing to give up in order to hold λ_t constant for all future periods with no changes to the consumption process. Defining $\widehat{U}_0$ as the utility of the consumption process with constant λ_t,

$$\widehat{\pi} = 1 - \frac{U_0}{\widehat{U}_0}, \tag{20}$$

where $\widehat{U}_0$ is calculated using the same numerical method described for U_0.

As reported in Table 2, the resulting valuation premium is 90% in the benchmark model and 55% in the extended model. Like the timing premia calculated in the previous subsection, these seem very high. Certainly, they suggest that preferences toward valuation risk are not as moderate as relative risk aversion of 1.5 to 2.4 would suggest in isolation.

The final row of Table 2 reports total risk premia, $\overline{\pi} = 1 - \frac{U_0}{\overline{U}_0}$, where $\overline{U}_0$ is utility with constant λ_t and consumption process $\overline{C}_t = \mathrm{E}_0 [C_t]$ for all t. Analogous to the timing and valuation risk premia, total risk premium represents that fraction of total and expected future consumption one would give up to avoid all risk. In the benchmark model, $\overline{\pi}$ is 90%, and $\overline{\pi}$ is 94% in the extended model, again indicating that the valuation risk model implies a high risk premium despite its seemingly modest coefficient of relative risk aversion.

3 Empirical assessment

To empirically assess the valuation risk model, I simulate the model and compare basic moments in the simulated and historical data. I then investigate what is driving the equity premium in the model. Like the long run risk model of Bansal and Yaron (2004), the valuation risk model features important correlations between stock returns and small but highly persistent shocks to long-term growth rates. Albuquerque et al. (2016) primarily focus on long term λ_t growth shocks, but as discussed in Section 1.2, the extended model also includes important shocks to long term consumption and dividend growth. Given the importance of these long-term shocks, I follow Beeler and Campbell's (2012) empirical assessment of the long-run risk model and ask whether the valuation risk model's growth and return persistence and predictability are consistent with historical data.

For historical data on consumption, dividends, and returns, I use the 1930 to 2008 U.S. annual data constructed and used by Beeler and Campbell (2012) and Bansal, Kiku, and Yaron (2012).[12] Consumption data is U.S. real nondurables and services consumption from the Bureau of Economic Analysis. Stock return and dividend data is from CRSP, converted to real values using the CPI. The ex-ante real risk-free rate comes from forecast regressions using current Treasury bill yields and lagged inflation. See Beeler and Campbell (2012) for additional details. Except for slight differences in their methodology for estimating ex-ante real risk-free rates, the data are equivalent to the U.S. data Albuquerque et al. (2016) construct for 1929 to 2011 with similar empirical sample moments.

To simulate the model, I generate 100,000 series of i.i.d. random variables ε^c_{t+1}, ε^d_{t+1}, $\varepsilon^\Lambda_{t+1}$, ω_{t+1}, and η_{t+1} to generate simulated monthly consumption and dividends based on

[12] I thank Jason Beeler and John Campbell for posting their data and associated code. For consistency, I use the same data without updates and follow Beeler and Campbell's (2012) empirical methodology as closely as possible.

the processes specified by equations (14) and (15). Monthly market returns and risk free rates are calculated based on return equations discussed in the Internet Appendix. Each simulation is initiated for 10 years and then generates 79 years of simulated data to match the length of the historical sample. One complexity is dealing with negative realizations of σ_t^2. While shocks to σ_t^2 are small relative to its long-term mean, they are persistent enough that negative realizations of σ_t^2 are reasonably common, occurring at some point during 76% of extended model simulations. I keep these negative realizations of σ_t^2 for purposes of calculating σ_{t+1}^2 and the $\alpha_c\left(\sigma_{t+1}^2-\sigma^2\right)$ and $\alpha_d\left(\sigma_{t+1}^2-\sigma^2\right)$ terms of Δc_{t+1} and Δd_{t+1}, but I replace σ_t with a small positive number for purposes of calculating $\sigma_t\varepsilon_{t+1}^c$, $\pi_{dc}\sigma_t\varepsilon_{t+1}^c$, and $\varphi\sigma_t\varepsilon_{t+1}^d$ whenever σ_t^2 is negative to ensure that consumption and dividend shock variance is always positive.[13] The simulated data is annualized following the conventions described by Beeler and Campbell (2012). Annual log returns are the sum of monthly log returns. For consumption and dividend growth, monthly consumption and dividends are summed, and annual growth is the growth rate of the sum. To annualize price dividend ratios, I multiply the year-end monthly price-dividend ratio by the last month's dividend and divide by the sum of all dividends over the past year.

3.1 Basic moments

Table **3** reports basic moments of the data along with median simulated moments from 100,000 simulations of the benchmark and extended models. For the most part, the simulated moments are similar to the model moments reported by Albuquerque et al. (2016).[14] The

[13]This approach differs from the approach taken by Bansal and Yaron (2004), Bansal, Kiku, and Yaron (2012), and Beeler and Campbell (2012), who instead replace all negative realizations of σ_t^2 with a small positive value. The distinction is meaningful because in the valuation risk model, expected consumption and dividend growth in part depend on σ_t^2. Thus, censoring σ_t^2 changes long-run average consumption and dividend growth. Albuquerque et al. (2016) are silent as to how they handle negative realizations of σ_t^2, but their reported model average consumption and dividend growth indicate that they retain negative realizations of σ_t^2.

[14]Two exceptions are consumption growth serial correlation and the mean risk-free rate. The difference in reported serial correlations appears to be due to methodology for annualizing consumption growth. Summing monthly log consumption growth instead of calculating the log of annual consumption growth generates serial correlations close to those reported by Albuquerque et al. (2016). Albuquerque et al. constrain their median

main takeaway from Table 3 is that the benchmark and extended models are both reasonably successful at matching means, standard deviations, and serial correlations of consumption growth, dividend growth, stock market returns, the risk-free rate, and the price-dividend ratio. This success is aided to some extent by the fact that the models were estimated based on many of these moments. Still, it is a victory for the models. With empirically reasonable consumption and dividend growth, the models match empirical stock return and price-dividend levels and volatility with moderate relative risk aversion of 1.5 in the benchmark model and 2.4 in the extended model.

[Insert Table 3 Here]

3.2 Equity premium

The primary result of the valuation risk model is that shocks to time preferences create valuation risk that can explain the observed equity premium with low risk aversion despite low correlation between stock returns and consumption growth. This result is easiest to see in the benchmark model, which generates an equity premium of 7.48% in the median simulation, compared to an average equity premium of 7.01% in the historical data.[15] By design, the benchmark model's equity premium is entirely driven by valuation risk. As reported in Table 4 and discussed by Albuquerque et al. (2016), shutting down valuation risk by setting $\sigma_\Lambda = \sigma_\eta = 0$ and re-simulating the economy under identical values for other parameters results in an equity premium of 0.01%.

[Insert Table 4 Here]

The extended model adds additional shocks to match other moments in the data. As a result, risk in the extended model is more complicated. In addition to valuation risk, the

risk-free rate to match the point estimate in the data, resulting in median risk-free rates of 0.13%, whereas the median simulated mean risk-free rates in Table 3 are -0.01% for the benchmark model and 0.33% for the extended model.

[15] The equity premium in both the data and the simulations is calculated as the mean log excess return plus one half of its variance.

model has time-varying conditional variance, which in turn affects expected consumption and dividend growth. Albuquerque et al. (2016) consider setting $\sigma_\Lambda = 0$ to shut down valuation risk and setting $\sigma_\omega = 0$ to shut down conditional volatility shocks and conclude that valuation risk and conditional volatility both play important roles in generating the equity premium. This decomposition overlooks the fact that shocks to σ_t^2 change both conditional variances and conditional expectations. The $\alpha_c\left(\sigma_{t+1}^2 - \sigma^2\right)$ and $\alpha_d\left(\sigma_{t+1}^2 - \sigma^2\right)$ terms in the consumption and dividend growth processes create persistent shocks to expected growth that function in much the same way as the persistent growth shocks in long-run risk models. Setting $\sigma_\omega = 0$ simultaneously shuts down both conditional volatility and persistent growth shocks.

To explore the relative importance of valuation risk, conditional volatility, and long-run risk created by persistent growth shocks, I consider shutting each channel down one step at a time. Table 4 reports the results. The extended model initially generates an equity premium of 6.28%. Shutting down valuation risk by setting $\sigma_\Lambda = \sigma_\eta = 0$ decreases the equity premium from 6.28% to 4.46%. I next shut down the long-run risk channel by setting $\alpha_c = \alpha_d = 0$. The equity premium drops to 3.14% with valuation risk and 0.01% without valuation risk. By contrast, shutting down conditional volatility by setting $\sigma_\omega = 0$ after the long-run risk channel has already been shut down has no impact on the equity premium.[16] The implication is that the equity premium in the extended valuation risk model heavily relies on long-run risk associated with persistent expected growth shocks. Instead of being an alternative to long-run risk, the extended valuation risk model is essentially a model of long-run risk supplemented with valuation risk.

[16]It would be interesting to consider shutting down conditional volatility while preserving the long-run risk channel. However, this is not possible because setting $\sigma_\omega = 0$ shuts down both conditional volatility and long-run risk.

3.3 Growth and return persistence

Because the valuation risk model involves persistent growth and time preference shocks, it is important to assess whether the model's persistence is consistent with the data. Beeler and Campbell (2012) propose assessing persistence by computing variance ratios over different horizons. The K-period variance ratio for consumption growth is:

$$\widehat{V}(K) = \frac{\widehat{Var}\left(\Delta c_{t+1} + ... + \Delta c_{t+K}\right)}{K\widehat{Var}\left(\Delta c_t\right)}. \tag{21}$$

Population variance ratios depend on weighted average population autocorrelations: $V(K) = 1 + 2\sum_{j=1}^{K-1}\left(1 - \frac{j}{K}\right)\rho_j$, where ρ_j is the correlation between Δc_t and Δc_{t+j}. I calculate variance ratios for consumption growth, dividend growth, and the risk-free rate in the data and simulations. Table **5** reports results for 2-, 4-, and 6-year variance ratios. As discussed by Beeler and Campbell, consumption and dividend growth have positive one-year autocorrelation followed by negative autocorrelations at longer horizons in the data. These long term reversions result in consumption and dividend growth variance ratios of less than one at a horizon of six years.

[Insert Table **5** Here]

The benchmark model's consumption and dividend growth persistence is generally consistent with the data. Across 100,000 simulations, the median consumption growth variance ratio is 1.23 at a 2-year horizon, 1.30 at a 4-year horizon, and 1.28 at a 6-year horizon. Compared to consumption growth variance ratios of 1.40, 1.38, and 0.84 in the data, the model generates slightly less short-term autocorrelation than the data and does not exhibit the longer-term reversion seen in the data. Nonetheless, these differences are modest and fall short of or close to 10% significance thresholds. Across the 100,000 consumption growth simulations, 4.3% of 2-year variance ratios, 37.4% of 4-year variance ratios, and 91.6% of 6-year variance ratios are higher than the data. Simulated dividend growth variance ratios are close to the data at a horizon of two years and are higher than the data at longer horizons, with a significant difference for the six-year variance ratio.

Results are less encouraging for the extended model, which generates large consumption and dividend growth persistence. The median 6-year variance ratio in the extended model is 3.28 for consumption growth and 2.61 for dividend growth, and 100% of simulations have 6-year variance ratios above that observed in the data, implying that 6-year variance ratios soundly reject the extended model. Not only are these variance ratios high relative to the data, they are also higher than the variance ratios generated by Bansal and Yaron's (2004) long-run risk model, which Beeler and Campbell (2012) calculate as 2.32 for consumption growth and 1.87 for dividend growth. As estimated, the extended valuation risk model is not an alternative to long-run risk models. Rather, it has even more persistent consumption and dividend growth than long-run risk models themselves do. As discussed in Section 1.2, persistent growth shocks are embedded in the valuation risk model through the dependence of consumption and dividend growth on persistent changes to volatility. The results in Table **5** indicate that this channel generates significant long run risk.

Panel C of Table **5** shows risk-free rate persistence. Valuation risk comes from persistent shocks to time preferences, which in turn shift the risk-free rate up or down. Is the resulting risk-free rate persistence consistent with the data? For the most part, the answer is yes. In the data, the risk-free rate variance ratio is 1.67 at a 2-year horizon, 2.45 at a 4-year horizon, and 3.03 at a 6-year horizon. The benchmark model generates higher risk-free rate variance ratios than this, but the extended model's risk-free rate variance ratios are close to those observed in the data, which indicates that the extended model does a reasonable job of replicating historical risk-free rate persistence.

3.4 Growth and return predictability

Price-dividend ratio fluctuations in the valuation risk model are primarily driven by persistent shocks to time preferences and conditional variance, which also impact expected consumption and dividend growth through the $\alpha_c\left(\sigma_{t+1}^2 - \sigma^2\right)$ and $\alpha_d\left(\sigma_{t+1}^2 - \sigma^2\right)$ terms in equation (15). In addition to impacting time preferences and expected growth rates, these shocks also affect the risk-free rate, equity premium, and asset prices. Thus, it is natural to assess how growth and return predictability in the model relate to predictability in the observed historical data. For this analysis, I again follow Beeler and Campbell (2012) and regress future returns and growth on the current price-dividend ratio at horizons of 1, 3, and 5 years.

Table **6** reports the results. The first three columns summarize regressions of log excess returns (Panel A), log consumption growth (Panel B), log dividend growth (Panel C), and log risk-free rates (Panel D) on log price-dividend ratios in the data. Panels A, B, and C are identical to Beeler and Campbell (2012), and Panel D applies the same methodology to risk-free rates. The regressions replicate the longstanding result that the price-dividend ratio predicts excess returns and does not predict consumption growth, dividend growth, or the real risk-free rate (Fama and French, 1988; Campbell and Shiller, 1988). The remainder of Table **6** reports results from the same regressions in 100,000 simulations of the benchmark and extended models. For each regression, the table reports the median simulated R^2 and the fraction of simulations with an R^2 greater than the R^2 of the data. Simulated β coefficients and percentiles are reported in Internet Appendix Table IA.1 with equivalent results.

[Insert Table **6** Here]

As reported in Panel A, the price-dividend ratio predicts future excess returns in the data, with growing predictive power as the horizon increases, resulting in an R^2 of 0.27 for the 5-year regression. As discussed by Albuquerque et al. (2016) with essentially the same regression analysis, the benchmark model fails to replicate this relation. Expected excess returns are constant in the benchmark valuation risk model, resulting in median simulated R^2 values close to zero, which are rejected by the data at long horizons. The extended model fares better. At a horizon of five years, the model's median simulated R^2 is 0.06. This is lower than the 0.27 R^2 in the data, but 7.5% of simulations generate a higher R^2 than the data.

Turning to consumption and dividend growth in Panels B and C, the price-dividend ratio does not predict consumption or dividend growth in the data, particularly at long horizons. The benchmark model shares this lack of predictability. However, the price-dividend ratio strongly predicts consumption and dividend growth in the extended model. For consumption growth, the median simulated 5-year R^2 is 0.54, and 99.9% of simulations have an R^2 greater than the data, implying that the data strongly rejects the model. Similarly, the median simulated 5-year R^2 for dividend growth is 0.48, and 96.5% of simulations have an R^2 greater than the data.

Panel D repeats the same regression analysis for risk-free rates. The price-dividend ratio does not predict future risk-free rates in the data. The benchmark model counterfactually generates strong risk-free rate predictability. This is not surprising. Time preference is the only state variable affecting the price-dividend ratio in the benchmark model. As Λ_{t+1} decreases, prices fall and expected future risk-free rates increase. This is the essence of valuation risk. Investor impatience increases discount rates, causing prices to fall. The benchmark model is too simple to expect it to match all features of the data. Nonetheless, it is noteworthy that this core feature of valuation risk is completely missing in the data. Instead of moving inversely with the expected future risk-free rate, stock prices are largely unrelated to the risk-free rate in the data.[17] If anything, the sign goes the wrong way. Albuquerque et al. (2016) conduct similar analysis on the contemporaneous correlation between stock prices and the risk-free rate and note that the relation varies across countries and is sensitive to the sample time period being analyzed. While this makes it difficult to reject valuation risk, it also highlights that there is limited support for valuation risk in the data.

The extended model moderates the negative correlation between the risk-free rate and

[17] Similarly, Campbell (1991) and Campbell and Ammer (1993) find that stock returns are highly related to news about expected excess returns, but news about the real risk-free rate has little impact on stock returns.

the price-dividend ratio by introducing a negative correlation between shocks to Λ_{t+1} and shocks to expected dividend growth through the $\pi_{d\Lambda}\varepsilon_{t+1}^{\Lambda}$ term in equation (15) with $\pi_{d\Lambda} < 0$. As Albuquerque et al. (2016) discuss, this change decreases the negative contemporaneous correlation between the risk-free rate and price-dividend ratio. Similarly, it eliminates the risk-free rate predictability in the regressions reported in Panel D of Table **6**, consistent with the data.

3.5 Valuation risk in the cross section

Is there evidence of a valuation risk premium in the cross section of stocks? If valuation risk generates a large risk premium, stocks with more exposure to valuation risk should earn higher average returns. A new literature on the term structure of equity returns suggests that this is unlikely to be the case. Binsbergen, Brandt, and Koijen (2012), Binsbergen and Koijen (2017), and Weber (2017) find longer duration stocks and claims to dividends further in the future have lower expected returns. These results are the opposite of what the valuation risk model would predict. Long duration claims are more sensitive to discount rate changes and should have higher valuation risk premia. Albuquerque et al. (2016) acknowledge this shortcoming and note that other asset pricing models are also inconsistent with this pattern. Nonetheless, given that duration is at the core of valuation risk, the growing evidence on the equity term structure would seem to be a problem for valuation risk. In the Internet Appendix, I further analyze cross-sectional valuation risk by sorting stocks based on their past return sensitivity to risk-free rate shocks and find no evidence that valuation risk is priced in the cross section of returns.

4 Conclusion

The valuation risk model is an important step forward for understanding how stochastic time preferences can affect asset prices. Epstein-Zin preferences applied to time preference shocks introduce a new source of risk for investors that could help to explain the level and volatility of stock prices observed in the data. However, the valuation risk model implies preferences toward valuation risk and resolution of uncertainty that are difficult to rationalize, and model predictions regarding long-run consumption and dividend growth are inconsistent with the data.

Empirical analysis highlights that the extended version of the valuation risk model embeds significant long-run consumption and dividend growth persistence and predictability that are not present in the data. This empirical inconsistency could be addressed by re-estimating the model with these moments, but decreasing the role of long-run risk would inevitably require valuation risk to explain more of the equity premium.

In the data, there is little relation between aggregate stock prices and the risk-free rate. If anything, the correlation goes the wrong way and stock prices increase with the risk-free rate. Thus, large aversion to valuation risk is required to generate a significant valuation risk premium. The Epstein-Zin preferences specified by equation (1) generate a large risk premium with relative risk aversion (γ) and elasticity of intertemporal substitution (ψ) that are reasonable in isolation. However, these parameters imply extreme preferences regarding resolution of uncertainty and valuation risk aversion. Extreme preferences are not necessarily a problem if the model is simply meant to explain the data, but if the goal is to explain the data with preferences that are quantitatively reasonable, it is important to understand what preferences the model actually implies by considering how γ and ψ interact with one another. This is true for models with Epstein-Zin utility in general and is particularly important when using Epstein-Zin utility to describe aversion to a new source of risk such as changing time preferences.

More generally, preference assessments highlight the potential complications of adding stochastic preferences to standard utility functions. More flexible utility functions may be necessary to separately describe and parameterize aversion to changing preferences. Developing such models is a difficult challenge but may be necessary for more progress to be made on stochastic preferences.

References

Albuquerque, Rui, Martin Eichenbaum, Victor Luo, and Sergio Rebelo, 2016, Valuation risk and asset pricing, *Journal of Finance* 71, 2861–2904.

Bansal, Ravi, Dana Kiku, and Amir Yaron, 2012, An empirical evaluation of the long-run risks model for asset prices, *Critical Finance Review* 1, 183–221.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Beaudry, Paul, and Eric van Wincoop, 1996, The intertemporal elasticity of substitution: An exploration using a US panel of state data, *Economica* 63, 495–512.

Beeler, Jason, and John Y. Campbell, 2012, The long-run risks model and aggregate asset prices: An empirical assessment, *Critical Finance Review* 1, 141–182.

Bhamra, Harjoat S., and Raman Uppal, 2014, Asset prices with heterogeneity in preferences and beliefs, *Review of Financial Studies* 27, 519–580.

Binsbergen, Jules H. v., Michael Brandt, and Ralph S. J. Koijen, 2012, On the timing and pricing of dividends, *American Economic Review* 102, 1596–1618.

Binsbergen, Jules H. v., and Ralph S.J. Koijen, 2017, The term structure of returns: Facts and theory, *Journal of Financial Economics* 124, 1–21.

Campbell, John Y., 1991, A variance decomposition for stock returns, *Economic Journal* 101, 157–179.

Campbell, John Y., 1993, Intertemporal asset pricing without consumption data, *American Economic Review* 487–512.

Campbell, John Y., 2003, Chapter 13: Consumption-based asset pricing, in *Financial Markets and Asset Pricing*, volume 1 of *Handbook of the Economics of Finance*, 803–887 (Elsevier).

Campbell, John Y., 2018, *Financial Decisions and Markets: A Course in Asset Pricing* (Princeton University Press).

Campbell, John Y., and John Ammer, 1993, What moves the stock and bond markets? A variance decomposition of long-term asset returns, *Journal of Finance* 48, 3–37.

Campbell, John Y., and John H. Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Campbell, John Y., and Robert J. Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.

Cochrane, John H., 2001, Presidential address: Discount rates, *Journal of Finance* 66, 1047–1108.

Creal, Drew, and Jing Cynthia Wu, 2020, Bond risk premia in consumption-based models, Working Paper, University of Notre Dame.

de Groot, Oliver, Alexander W. Richter, and Nathaniel A. Throckmorton, 2018, Uncertainty shocks in a model of effective demand: Comment, *Econometrica* 86, 1513–1526.

de Groot, Oliver, Alexander W. Richter, and Nathaniel A. Throckmorton, 2020, Valuation risk revalued, Working Paper, University of Liverpool Management School.

Dew-Becker, Ian, and Stefano Giglio, 2016, Asset pricing in the frequency domain: theory and empirics, *Review of Financial Studies* 29, 2029–2068.

Epstein, Larry G., Emmanuel Farhi, and Tomasz Strzalecki, 2014, How much would you pay to resolve long-run risk?, *American Economic Review* 104, 2680–2697.

Epstein, Larry G., and Stanley E. Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Epstein, Larry G., and Stanley E. Zin, 1991, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis, *Journal of Political Economy* 263–286.

Fama, Eugene F., and Kenneth R. French, 1988, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3–25.

Garleanu, Nicolae, and Stavros Panageas, 2015, Young, old, conservative, and bold: The implications of heterogeneity and finite lives for asset pricing, *Journal of Political Economy* 123, 670–685.

Gruber, Jonathan, 2013, A tax-based estimate of the elasticity of intertemporal substitution, *Quarterly Journal of Finance* 03, 1–20.

Hall, Robert E, 1988, Intertemporal substitution in consumption, *Journal of Political Economy* 96, 339–357.

Hansen, Lars P., and Thomas J. Sargent, 2008, *Robustness* (Princeton University Press).

Kreps, David M., and Evan L. Porteus, 1978, Temporal resolution of uncertainty and dynamic choice theory, *Econometrica* 46, 185–200.

Maurer, Thomas, 2012, Is consumption growth merely a sideshow in asset pricing?, Working Paper, Washington University in St. Louis.

Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.

Schorfheide, Frank, Dongho Song, and Amir Yaron, 2018, Identifying long-run risks: a Bayesian mixed-frequency approach, *Econometrica* 86, 617–654.

Uhlig, Harald, 2014, Discussion of valuation risk and asset pricing, Conference on Aggregate Fluctuations: Causes and Consequences, http://www.csef.it/IMG/pdf/discussion_uhlig.pdf.

Weber, Michael, 2017, Cash flow duration and the term structure of equity returns, *Journal of Financial Economics* 128, 486–503.

Weil, Philippe, 1989, The equity premium puzzle and the risk-free rate puzzle, *Journal of Monetary Economics* 24, 401–421.

Table 1. Parameter values

Description: This table reports parameter values for the valuation risk model described by equations (14) and (15) as specified and estimated by Albuquerque et al. (2016).
Interpretation: These parameter values describe the benchmark and extended models.

Parameter	Benchmark Model	Extended Model	Parameter	Benchmark Model	Extended Model
γ	1.5160	2.3961	ρ_d	0	0.33427
ψ	1.4567	2.2107	α_d	0	-151.881
θ (implied)	-1.6458	-2.5492	φ	2.478552	0.025274
δ	0.99795	0.99796	π_{dc}	0.071194	-1.3205
μ_c	0.0015644	0.0010428	$\pi_{d\lambda}$	0	-0.01091
ρ_c	0	0.094708	ρ_λ	0.99132	0.99168
α_c	0	-95.3767	σ_λ	0.000586	0.000386
σ	0.0069004	0.0046033	σ_η	0	0.007783
$\pi_{c\lambda}$	0	-0.0029185	ν	0	0.99717
μ_d	0.0015644	0.0007669	σ_ω	0	1.68E-06

Table 2. Timing and risk premia in the valuation risk model

Description: This table reports timing and risk premia for the valuation risk model. Timing premium is the maximum percent of current and future consumption agents would be willing to give up to resolve all future uncertainty at time 1. Valuation risk premium is the maximum percent of current and future consumption agents would be willing to give up to avoid valuation risk by holding λ_t constant for all t. Total risk premium is the maximum percent of current and future consumption agents would be willing to give up to avoid all risk by holding λ_t constant for all t and by changing the consumption process to a known endowment equal to the mean consumption endowment at each time implied by the original consumption process.
Interpretation: The valuation risk model embeds large timing and valuation risk premia.

	Benchmark model	Extended model
Timing premium (π^*)	82.3%	54.7%
Valuation risk premium ($\widehat{\pi}$)	89.6%	55.2%
Total risk premium ($\overline{\pi}$)	90.0%	93.6%

Table 3. Basic moments of the data and model

Description: This table reports means ($E()$), standard deviations ($\sigma()$), and autocorrelations ($\rho()$) in the historical data and model simulations. Log consumption growth is Δc. Log dividend growth is Δd. The log aggregate stock market return is r_m. The log real risk-free rate is r_f. The log price dividend ratio is $p - d$. Historical data is 1930 to 2008 annual data from the Bureau of Economic analysis and CRSP. The reported model moments are median values from 100,000 simulations with time periods equal to the historical data. The model is simulated monthly and then annualized for comparability to the data.
Interpretation: The valuation risk model is reasonably successful at matching means, standard deviations, and serial correlations of consumption growth, dividend growth, stock market returns, the risk-free rate, and the price-dividend ratio.

Moment	Data	Benchmark Model	Extended Model	Moment	Data	Benchmark Model	Extended Model
$E(\Delta c)$	1.93	1.88	1.39	$E(r_f)$	0.56	-0.01	0.33
$\sigma(\Delta c)$	2.16	1.94	2.49	$\sigma(r_f)$	2.89	4.41	4.10
$\rho(\Delta c)$	0.45	0.23	0.61	$\rho(r_f)$	0.65	0.89	0.50
$E(\Delta d)$	1.15	1.88	1.40	$E(p-d)$	3.36	3.23	3.42
$\sigma(\Delta d)$	11.05	4.81	6.56	$\sigma(p-d)$	0.45	0.32	0.51
$\rho(\Delta d)$	0.21	0.23	0.48	$\rho(p-d)$	0.87	0.84	0.90
$E(r_m)$	5.47	5.88	4.70				
$\sigma(r_m)$	20.17	17.65	19.45				
$\rho(r_m)$	0.02	-0.04	0.00				

Table 4. Equity premium in the valuation risk model

Description: Reported equity premia are median values from 100,000 simulations. Equity premia are calculated as mean annual log excess equity returns plus one half of its variance. Baseline simulations use parameter values reported in Table 1. Other simulations use baseline parameter values with the noted changes to reflect shutting down different combinations of valuation risk, long-run risk, and stochastic volatility.
Interpretation: The equity premium in the extended model heavily depends on long-run risk from persistent expected growth shocks in addition to valuation risk.

		Extended model	
	Benchmark model	Stochastic volatility	Homoscedastic ($\sigma_\omega = 0$)
Baseline	7.48	6.28	
No valuation risk ($\sigma_\lambda = \sigma_\eta = 0$)	0.01	4.46	
No long-run risks ($\alpha_c = \alpha_c = 0$)		3.14	3.13
No valuation or long-run risks ($\sigma_\lambda = \sigma_\eta = \alpha_c = \alpha_c = 0$)		0.01	0.01

Table 5. Variance ratios

Description: Variance ratios are calculated as $\widehat{V}(K) = \frac{\widehat{Var}(X_{t+1}+...+X_{t+K})}{K\widehat{Var}(X_t)}$ where X is annual log consumption growth, log dividend growth, or the log real risk-free rate. Variance ratios in the data are based on 1930 to 2008 annual historical data from the Bureau of Economic analysis and CRSP. Simulated variance ratios are based on 100,000 simulations with time periods equal to the historical data. The models are simulated monthly and then annualized for comparability with the historical data. For the simulations, the table reports median variance ratios and the % of simulated variance ratios that are larger than the comparable variance ratio observed in the data.
Interpretation: Variance ratios summarize the persistence of shocks to consumption growth, dividend growth, and the risk-free rate in the data and model.

		Benchmark model		Extended model	
	Data	Median	% > data	Median	% > data
Panel A. Consumption variance ratios					
2 years	1.40	1.23	4.3%	1.61	89.2%
4 years	1.38	1.30	37.4%	2.54	98.8%
6 years	0.84	1.28	91.6%	3.28	100.0%
Panel B. Dividend variance ratios					
2 years	1.23	1.23	51.5%	1.49	96.7%
4 years	0.98	1.30	90.7%	2.13	99.8%
6 years	0.59	1.28	99.3%	2.61	100.0%
Panel C. Risk-free rate variance ratios					
2 years	1.67	1.90	99.7%	1.51	13.8%
4 years	2.45	3.42	99.4%	2.36	42.9%
6 years	3.03	4.61	98.0%	3.03	50.1%

Table 6. Predictive regressions

Description: This table reports results from regressing future log excess equity returns (panel A), consumption growth (panel B), dividend growth (panel C), and real risk-free rates (panel D) on the current log price-dividend ratio. Regressions in the data are based on 1930 to 2008 annual historical data from the Bureau of Economic analysis and CRSP. Simulation regressions are based on 100,000 simulations with time periods equal to the historical data. The models are simulated monthly and then annualized for comparability with the historical data. For the simulations, the table reports the median R^2 and the % of simulated R^2 that are larger than the comparable regression R^2 in the data. Standard errors are Newey-West with 2*(horizon-1) lags.
Interpretation: Predictive regressions show the extent to which the price-dividend ratios predicts subsequent returns, consumption growth, dividend growth, and risk-free rates in the data and model.

	Data			Benchmark model R^2		Extended model R^2	
	$\hat{\beta}$	t	R^2	Median	% > data	Median	% > data
Panel A. Excess returns							
1 year	-0.09	-1.80	0.04	0.01	10.8%	0.01	15.6%
3 years	-0.26	-3.23	0.17	0.02	4.7%	0.04	8.1%
5 years	-0.41	-3.78	0.27	0.04	3.8%	0.06	7.5%
Panel B. Consumption growth							
1 year	0.01	1.59	0.06	0.01	6.9%	0.39	96.4%
3 years	0.01	0.59	0.01	0.02	57.7%	0.53	99.4%
5 years	0.00	-0.06	0.00	0.03	96.8%	0.54	99.9%
Panel C. Dividend growth							
1 year	0.07	1.98	0.09	0.01	4.0%	0.26	85.6%
3 years	0.11	1.33	0.06	0.02	22.5%	0.43	95.9%
5 years	0.09	1.21	0.04	0.03	42.3%	0.48	96.5%
Panel D. Risk-free rate							
1 year	0.01	1.15	0.03	0.91	100.0%	0.05	62.6%
3 years	0.03	0.82	0.03	0.74	100.0%	0.07	68.3%
5 years	0.05	1.06	0.05	0.61	100.0%	0.08	59.0%

Internet Appendix for:

"High Aversion to Stochastic Time Preference Shocks and Counterfactual Long-Run Risk in the Albuquerque et al. Valuation Risk Model"

A Solution details

This section of the appendix provides details and derivations for results discussed in the main text of the paper.

A.1 General pricing equations

The representative agent has the augmented Epstein-Zin preferences described by equation (1):

$$U_t = \left[\lambda_t C_t^{1-1/\psi} + \delta \left(\mathrm{E}_t \left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{1/(1-1/\psi)}.$$

Optimization is subject to a budget constraint of

$$W_{t+1} = R_{w,t+1} \left(W_t - C_t\right) \tag{IA.1}$$

where W_t is wealth at time t and $R_{w,t+1}$ is the return on the overall wealth portfolio, which is a claim to all future consumption.

Albuquerque et al. (2016) use standard techniques from the Epstein-Zin preference literature to show that the preferences represented by equation (1) imply the log stochastic discount factor expressed by equation (2):

$$m_{t+1} = \theta \log(\delta) + \theta \Lambda_{t+1} - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)\, r_{w,t+1}.$$

This is the same as the standard Epstein-Zin stochastic discount factor except that discounting is time-varying (i.e., $\delta \frac{\lambda_{t+1}}{\lambda_t}$ instead of δ).

Using $0 = \mathrm{E}_t\left[m_{t+1} + r_{i,t+1}\right] + \frac{1}{2}\left(\sigma_m^2 + \sigma_i^2 + 2\sigma_{mi}\right)$ (the log version of $1 = \mathrm{E}_t\left[M_{t+1}R_{i,t+1}\right]$), the expected return for any asset can be expressed as

$$\begin{aligned}\mathrm{E}_t\left[r_{i,t+1}\right] + \frac{1}{2}\sigma_i^2 &= -\theta\log\left(\delta\frac{\lambda_{t+1}}{\lambda_t}\right) + \frac{\theta}{\psi}\mathrm{E}_t\left[\Delta c_{t+1}\right] + (1-\theta)\,\mathrm{E}_t\left[r_{w,t+1}\right] \\ &\quad -\frac{1}{2}\left(\frac{\theta}{\psi}\right)^2\sigma_c^2 - \frac{1}{2}(1-\theta)^2\sigma_w^2 + \frac{\theta}{\psi}(\theta-1)\,\sigma_{wc} \\ &\quad +\frac{\theta}{\psi}\sigma_{ic} + (1-\theta)\,\sigma_{iw}. \qquad \text{(IA.2)}\end{aligned}$$

The $\frac{1}{2}\sigma_i^2$ on the left hand side of equation (IA.2) is the Jensen's inequality correction for log returns.

The resulting risk-free rate is

$$\begin{aligned}r_{f,t+1} &= -\theta\log\left(\delta\frac{\lambda_{t+1}}{\lambda_t}\right) + \frac{\theta}{\psi}\mathrm{E}_t\left[\Delta c_{t+1}\right] + (1-\theta)\,\mathrm{E}_t\left[r_{w,t+1}\right] \\ &\quad -\frac{1}{2}\left(\frac{\theta}{\psi}\right)^2\sigma_c^2 - \frac{1}{2}(1-\theta)^2\sigma_w^2 + \frac{\theta}{\psi}(\theta-1)\,\sigma_{wc}. \qquad \text{(IA.3)}\end{aligned}$$

Differencing equations (IA.2) and (IA.3) yields the risk premia of equation (6):

$$\mathrm{E}_t\left[r_{i,t+1}\right] - r_{f,t+1} + \frac{1}{2}\sigma_i^2 = \frac{\theta}{\psi}\sigma_{ic} + (1-\theta)\,\sigma_{iw},$$

which is exactly the same expression as in standard Epstein-Zin models. Substituting $\mathrm{E}_t\left[r_{w,t+1}\right]$ into equation (IA.3), yields equation (5):

$$r_{f,t+1} = -\log(\delta) - \Lambda_{t+1} + \frac{1}{\psi}\mathrm{E}_t\left[\Delta c_{t+1}\right] - \frac{1-\theta}{2}\sigma_w^2 - \frac{\theta}{2\psi^2}\sigma_c^2,$$

which is the same as standard Epstein-Zin models except that δ is replaced by $\delta\frac{\lambda_{t+1}}{\lambda_t}$.

A.2 Intertemporal CAPM

Following Campbell (1993), the budget constraint can be log-linearized to generate equation (7):

$$r_{w,t+1} - \mathrm{E}_t\left[r_{w,t+1}\right] = \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=0}^{\infty} \rho^j \Delta c_{t+1+j} - \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{\infty} \rho^j r_{w,t+1+j}$$

where $\rho = 1 - \exp\left(\overline{c-w}\right)$ is a log-linearization constant ($\overline{c-w}$ is the average log consumption-wealth ratio). Rearranging, current consumption shocks can be expressed as

$$\begin{aligned} \Delta c_{t+1} - \mathrm{E}_t\left[\Delta c_{t+1}\right] &= r_{w,t+1} - \mathrm{E}_t\left[r_{w,t+1}\right] \\ &\quad + \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{\infty} \rho^j r_{w,t+1+j} \\ &\quad - \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j}. \end{aligned} \tag{IA.4}$$

So far, we have only made use of modified Epstein-Zin preferences and the budget constraint. We now use assumptions about consumption and time preference innovations. Due to our homoscedasticity assumption, risk premia do not change over time, and the risk-free rate only changes in response to time preference and consumption growth innovations. Thus, innovations to expected returns can be decomposed as

$$\begin{aligned} \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) r_{w,t+1+j} &= \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) r_{f,t+1+j} \\ &= \left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \log\left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}}\right) + \frac{1}{\psi}\left(\mathrm{E}_{t+1} - \mathrm{E}_t\right)\left[\Delta c_{t+j+1}\right] \end{aligned} \tag{IA.5}$$

for $j \geq 1$. Substituting equation (IA.5) into equation (IA.4) yields

$$\begin{aligned} \Delta c_{t+1} - \mathrm{E}_t\left[\Delta c_{t+1}\right] &= r_{w,t+1} - \mathrm{E}_t\left[r_{w,t+1}\right] \\ &\quad - \left(1 - \frac{1}{\psi}\right)\left(\mathrm{E}_{t+1} - \mathrm{E}_t\right) \sum_{j=1}^{\infty} \rho^j \Delta c_{t+1+j} \end{aligned}$$

$$+\left(\mathrm{E}_{t+1}-\mathrm{E}_{t}\right)\sum_{j=1}^{\infty}\rho^{j}\log\left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}}\right). \quad \text{(IA.6)}$$

Substituting out consumption shock covariance (σ_{ic}) from equation (6) yields risk premia as a function of covariance with market returns and innovations to future time preferences and consumption growth:

$$\begin{aligned}\mathrm{E}_{t}\left[r_{i,t+1}\right]-r_{f,t+1}+\frac{1}{2}\sigma_{i}^{2} &= \gamma\sigma_{iw}+(\gamma-1)\frac{1}{\psi}cov_{t}\left(r_{i,t+1},\left(\mathrm{E}_{t+1}-\mathrm{E}_{t}\right)\sum_{j=1}^{\infty}\rho^{j}\Delta c_{t+1+j}\right)\\ &\quad+\frac{\theta}{\psi}cov_{t}\left(r_{i,t+1},\left(\mathrm{E}_{t+1}-\mathrm{E}_{t}\right)\sum_{j=1}^{\infty}\rho^{j}\log\left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}}\right)\right). \quad \text{(IA.7)}\end{aligned}$$

This can be alternatively expressed as

$$\mathrm{E}_{t}\left[r_{i,t+1}\right]-r_{f,t+1}+\frac{1}{2}\sigma_{i}^{2}=\gamma\sigma_{iw}-\frac{\gamma-1}{\psi-1}\sigma_{ih(\lambda)}+(\gamma-1)\,\sigma_{ih(c)} \quad \text{(IA.8)}$$

where

$$\sigma_{ih(\lambda)}=cov_{t}\left(r_{i,t+1},\left(\mathrm{E}_{t+1}-\mathrm{E}_{t}\right)\sum_{j=1}^{\infty}\rho^{j}\log\left(\frac{\lambda_{t+j}}{\lambda_{t+j+1}}\right)\right)$$

and

$$\sigma_{ih(c)}=\frac{1}{\psi}cov_{t}\left(r_{i,t+1},\left(\mathrm{E}_{t+1}-\mathrm{E}_{t}\right)\sum_{j=1}^{\infty}\rho^{j}\Delta c_{t+1+j}\right)$$

are the two different types of risk-free rate news covariance.

Equation (IA.8) is an intertemporal capital asset pricing model (ICAPM) pricing equation. As in Campbell (1993), risk premia are a function of covariance with the market return and covariance with shocks to investment opportunities. Market return risk (σ_{iw}) is priced by relative risk aversion (γ) as in other ICAPM models. Also consistent with other ICAPM models, future interest rate covariance ($\sigma_{ih(c)}$ and $\sigma_{ih(\lambda)}$) is priced only if $\gamma \neq 1$. Yet, the two components of interest rate risk have different prices. Whereas $\sigma_{ih(c)}$ is priced by $\gamma-1$, $\sigma_{ih(\lambda)}$ is priced by $-\frac{\gamma-1}{\psi-1}$. When $\psi>1$, the prices have opposite signs, and if ψ is close to 1, time-preference risk is amplified relative to consumption growth risk. The key distinction between equation (IA.8) and more standard ICAPM models such as Campbell (1993) is that equation (IA.8) includes shocks to both consumption growth and time preferences. Because Campbell (1993) assumes preferences are constant, there is no $\sigma_{ih(\lambda)}$ in his model, and σ_{ih} is equivalent to $\sigma_{ih(c)}$.

A.3 Extended consumption CAPM

The budget constraint can also be used to substitute out wealth portfolio return covariance (σ_{iw}) from equation (6) by rearranging equation (IA.6) and using it to decompose σ_{iw}, thereby yielding equation (9):

$$\mathrm{E}_t\left[r_{i,t+1}\right] - r_{f,t+1} + \frac{1}{2}\sigma_i^2 = \gamma\sigma_{ic} + \left(\gamma\psi - 1\right)\sigma_{ih(c)} - \frac{\gamma\psi - 1}{\psi - 1}\sigma_{ih(\lambda)}.$$

A.4 Augmented consumption

Another way to derive the ICAPM and extended CCAPM pricing equations is to change notation to consider time preference shocks in the same units as consumption. Specifically, consider augmented consumption, defined as

$$\widetilde{C}_t \equiv \lambda_t^{1/(1-1/\psi)} C_t. \tag{IA.9}$$

With this notation change, equation (1) is transformed into standard Epstein-Zin preferences with respect to augmented consumption. All of Campbell's (1993) and Bansal and Yaron's (2004) results hold with respect to augmented consumption and returns measured in units of augmented consumption. In particular, the augmented risk-free rate is

$$\widetilde{r}_{f,t+1} = -\log\left(\delta\right) + \frac{1}{\psi}\mathrm{E}_t\left[\Delta\widetilde{c}_{t+1}\right] - \frac{1-\theta}{2}\sigma_w^2 - \frac{\theta}{2\psi^2}\sigma_c^2 \tag{IA.10}$$

and the risk premium for any asset is given by

$$\mathrm{E}_t\left[\widetilde{r}_{i,t+1}\right]-\widetilde{r}_{f,t+1}+\frac{1}{2}\sigma_i^2=\gamma\sigma_{iw}+(\gamma-1)\,\sigma_{ih(\widetilde{c})} \tag{IA.11}$$

where tildes represent augmented consumption and returns. Using the identities $\widetilde{r}_{i,t+1} = r_{i,t+1}+\frac{1}{1-1/\psi}\log\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$ and $\Delta\widetilde{c}_{t+1}=\Delta c_{t+1}+\frac{1}{1-1/\psi}\log\left(\frac{\lambda_{t+1}}{\lambda_t}\right)$, equations (IA.10) and (IA.11) are equivalent to equations (5) and (IA.8).

A.5 Calibrated model solution

Albuquerque et al. (2016) solve the model using log-linear analytical approximations. Let portfolio w be the overall wealth portfolio, which represents a claim to aggregate consumption. Using Campbell and Shiller's (1988) approximation for the return on the overall wealth portfolio the log return to the wealth portfolio can be expressed as

$$r_{w,t+1}=\kappa_0+\kappa_1 z_{t+1}-z_t+\Delta c_{t+1} \tag{IA.12}$$

where z_t is the log wealth-consumption ratio at time t. Unknown linearization parameters κ_0 and κ_1 are given by

$$\kappa_1 = \frac{\exp(z)}{1+\exp(z)} \tag{IA.13}$$

$$\kappa_0 = \log\left(1+\exp(z)\right)-\kappa_1 z \tag{IA.14}$$

where z is the unconditional mean of z_t. Returns to the market portfolio, which is a claim to aggregate dividends, can be similarly approximated as

$$r_{m,t+1}=\kappa_{m0}+\kappa_{m1}z_{m,t+1}-z_{m,t}+\Delta d_{t+1} \tag{IA.15}$$

with unknown parameters κ_{m0} and κ_{m1} constructed the same way.

Albuquerque et al. (2016) guess and verify that z_t and $z_{m,t}$ linearly depend on state variables, taking the form

$$z_t = A_0 + A_1 x_t + A_2 \eta_{t+1} + A_3 \sigma_t^2 + A_4 \Delta c_t \tag{IA.16}$$

$$z_{m,t} = A_{m0} + A_{m1} x_t + A_{m2} \eta_{t+1} + A_{m3} \sigma_t^2 + A_{m4} \Delta c_t + A_{m5} \Delta d_t, \tag{IA.17}$$

and solve for the unknown coefficients as functions of the model parameters and κ_0, κ_1, κ_{m0}, and κ_{m1}, which are functions of z and z_m. Closed-form solutions for these coefficients are reported in Albuquerque et al.'s internet appendix. One can then numerically iterate to find fixed points for z and z_m. Having solved for all coefficients, market returns in any period are given by equation (IA.15). To complete the solution, Albuquerque et al. use the stochastic discount factor (equation (2)), Euler equation, and equation (IA.12) to obtain the risk-free rate as a function of state variables.

B Valuation risk in the cross section

I analyze cross-sectional valuation risk by sorting stocks based on their past return sensitivity to risk-free rate shocks. Ideally, we would like to separately measure consumption growth and time preference risk-free rate shocks. Given the unobservability of time preferences and the imprecise and low-frequency nature of consumption data, measuring aggregate risk-free rate shocks is probably the best we can do. While this does not formally test the model, it assesses whether there is support in the cross section for valuation risk. If exposure to risk-free rate shocks is not priced in the cross-section, this suggests that valuation risk is not a major factor for explaining asset prices. The model informs how we measure risk-free rate shocks. In particular, it highlights that investors care about shocks to both current and expected future risk-free rates. Thus, instead of considering just $cov_t\left(r_{i,t+1}, r_{f,t+2} - \mathrm{E}_t\left[r_{f,t+2}\right]\right)$, I focus on $\sigma_{ih} = cov_t\left(r_{i,t+1}, (\mathrm{E}_{t+1} - \mathrm{E}_t)\sum_{j=1}^{\infty}\rho^j r_{w,t+1+j}\right)$.

To assess sensitivity to valuation shocks, we need to estimate $(\mathrm{E}_{t+1} - \mathrm{E}_t)\sum_{j=1}^{\infty}\rho^j r_{f,t+1+j}$.

This estimation has two challenges. First, the focus is on real interest rates. This is the risk-free rate in the model, and it is the relevant quantity for economic decisions. Unfortunately, the real risk-free rate is not directly observable. To overcome this challenge, I model expected Consumer Price Index (CPI) inflation and estimate the monthly real risk-free rates as the difference between the nominal 1-month Treasury bill yield and expected inflation over the next month. Baseline estimates focus on the 1983 to 2012 time period because monetary policy is more consistent and inflation is less volatile during this period than in previous periods.

A second empirical challenge is that valuation risk involves shocks to expectations. Thus, we need to estimate interest rate expectations. I do this with a vector autoregression (VAR) of interest rates, inflation, and other state variables. From the VAR, we extract an estimate for the time series of $(\mathrm{E}_{t+1} - \mathrm{E}_t) \sum_{j=1}^{\infty} \rho^j r_{f,t+1+j}$ innovations, which I then use to estimate σ_{ih} for individual stocks.

B.1 Vector autoregression

The VAR model is

$$Y_t = AY_{t-1} + \epsilon_t. \tag{IA.18}$$

Y_t is a $k \times 1$ vector with the nominal 1-month Treasury bill log yield and seasonally adjusted log CPI inflation over the past month as its first two elements. The remaining elements of Y_t are state variables useful for forecasting these two variables. The assumption that the VAR model has only one lag is not restrictive because lagged variables can be included in Y_t. Before estimating the VAR, Y_t is demeaned to avoid the need for a constant in equation (IA.18).

Vector ei is defined to be the ith column of a $k \times k$ identity matrix. Using this notation, expectations and shocks to current and future expectations can be extracted from Y_t, A, and ϵ_t. The real risk-free interest rate is estimated as the nominal 1-month Treasury bill yield less expected inflation:

$$\widehat{r_{f,t+1}} = (e1' - e2'A)\, Y_t. \tag{IA.19}$$

Similarly, expected future risk-free rates are

$$\mathrm{E}_t \left[\widehat{r_{f,t+j}}\right] = (e1' - e2'A)\, A^{j-1} Y_t. \tag{IA.20}$$

Shocks to current and expected risk-free rates are

$$(\mathrm{E}_{t+1} - \mathrm{E}_t)\, \widehat{r_{f,t+1+j}} = (e1' - e2'A)\, A^{j-1} \epsilon_{t+1}. \tag{IA.21}$$

Total interest rate news is

$$\begin{aligned} News_{h,t+1} &= (\mathrm{E}_{t+1} - \mathrm{E}_t) \sum_{j=1}^{\infty} \rho^j \widehat{r_{f,t+1+j}} \\ &= (e1' - e2'A) \sum_{j=1}^{\infty} \rho^j A^{j-1} \omega_{t+1} \\ &= (e1' - e2'A)\, \rho\, (I - \rho A)^{-1}\, \omega_{t+1} \end{aligned} \tag{IA.22}$$

where I is the identity matrix and ρ is a log linearization coefficient equal to $1 - \exp\left(\overline{c - w}\right)$ where $\overline{c - w}$ is the average log consumption-wealth ratio. I use a monthly coefficient value of $\rho = 0.996$ for the analysis.

To select state variables to include in Y_t, I first follow Campbell (1996) and include the relative Treasury bill rate, defined as the difference between the current one-month Treasury bill yield and the average one-month Treasury bill yield over the previous 12 months. I also include the relative monthly CPI inflation rate, defined the same way. Next, I include the yield spread between 10-year Treasury bonds and 3-month Treasury bonds because the slope of the yield curve is known to predict interest rate changes. Finally, I include the CRSP value-weighted market return and the log dividend-price ratio (defined as dividends over the past year divided by current price), which is known to predict market returns.

These variables are useful to the extent that equity returns are related to expected future interest rates. Equation (IA.18) can also be estimated with lags of Y_t. Because the Bayesian Information Criteria is insensitive to adding lags, I do not include lagged variables in Y_t.

Table IA.2 reports coefficient estimates and standard errors for the elements of A related to predicting nominal interest rates and inflation. The first two columns report results for the 1983 to 2012 time period, which is the primary focus. Nominal interest rate shocks are highly persistent with lag coefficient of 0.96. Inflation shocks are much less persistent and have a lag coefficient of 0.07. Inflation is increasing in lagged nominal yields. The VAR explains 95% of the variation in nominal yields over time. Inflation changes are less predictable with an R-squared of 0.24.

Figure IA.1 plots the estimated real risk-free rate from the VAR model along with the nominal one-month Treasury bill yield and the Federal Reserve Bank of Cleveland's real risk-free rate estimate.[1] As one would expect in a stable inflation environment, real interest rates generally follow the same pattern as nominal interest rates. Nonetheless, inflation expectations do change over time, particularly late in the sample. The VAR real risk-free rate estimate closely tracks the Federal Reserve Bank of Cleveland's estimate.

As a robustness check, I also estimate real risk-free rates and real risk-free rate news over a longer time period, starting in 1927. The methodology for the longer time period is the same as before except that the CPI is unadjusted because the seasonally adjusted CPI is only available starting in 1947. Columns (3) and (4) of Table IA.2 report the VAR results. In the extended time sample, inflation shocks are more persistent (inflation's lagged coefficient is 0.78, compared to 0.07 before). The results are otherwise similar to the original VAR.

B.2 Cross-sectional results

To assess whether valuation risk is priced in the cross section, I sort stocks into portfolios based on past covariance with risk-free rate news. Risk-free rate news covariance,

[1] The Federal Reserve Bank of Cleveland's real risk-free rate estimates are described by Haubrich, Pennacchi, and Ritchken (2008, 2012).

$\sigma_{ih} = cov_t\left(r_{i,t+1}, (\mathrm{E}_{t+1} - \mathrm{E}_t)\sum_{j=1}^{\infty}\rho^j r_{w,t+1+j}\right)$, is estimated on a rolling basis for all NYSE, AMEX, and NASDAQ common stocks using returns and risk-free rate news over the past three years, with the requirement that included stocks must have at least two years of historical data. Value-weighted decile portfolios are formed monthly by sorting stocks according to those estimates.

Table IA.**3** reports market capitalization, average excess returns, and $\beta_{ih} = \frac{\sigma_{ih}}{\sigma_h^2}$ estimates for each portfolio. The table also reports pricing errors (alphas) relative to the CAPM and Fama and French (1993) three factor model and factor loadings (betas) for the three factor model. Panel A reports results for the baseline 1985-2012 time period.[2] Risk-free rate news betas increase across the portfolios, and decile 10's news beta is a significant 0.58 higher than decile 1's news beta. Monthly excess returns are 42 bps lower in the 10th decile than in the 1st decile, but this return difference is not statistically significant, and there is no clear pattern to excess returns across the decile portfolios other than a drop in returns in decile 10. CAPM and 3 Factor alphas follow the same basic pattern. Factor loadings are also similar across the portfolios. The one exception is that decile 10 has a large negative loading on the value factor (HML). In short, there is no evidence that valuation risk is priced in the cross section of stock returns.

Results are similar in the extended 1929-2012 sample, reported in Panel B. Once again, average excess returns and alpha estimates decrease with interest rate news exposure, but the differences are not significant. The biggest difference between Panel A and Panel B is that β_{ih} differences across the portfolios are not significant in the extended sample. This suggests that stock-level valuation risk was not stable over time early in the sample, undercutting our ability to form valuation risk portfolios.

[2]Portfolio formation is based on at least two years of historical data, which causes the sample to start in 1985 instead of 1983.

Appendix References

Albuquerque, Rui, Martin Eichenbaum, Victor Luo, and Sergio Rebelo, 2016, Valuation risk and asset pricing, *Journal of Finance* 71, 2861–2904.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Campbell, John Y., 1993, Intertemporal asset pricing without consumption data, *American Economic Review* 487–512.

Campbell, John Y., 1996, Understanding risk and return, *Journal of Political Economy* 298–345.

Campbell, John Y., and Robert J. Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.

Fama, Eugene F., and Kenneth R. French, 1993, Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Haubrich, Joseph, George Pennacchi, and Peter Ritchken, 2008, Estimating real and nominal term structures using treasury yields, inflation, inflation forecasts, and inflation swap rates, Working Paper, Federal Reserve Bank of Cleveland.

Haubrich, Joseph, George Pennacchi, and Peter Ritchken, 2012, Inflation expectations, real rates, and risk premia: evidence from inflation swaps, *Review of Financial Studies* 25, 1588–1629.

C Supplemental tables and figure

Table IA.1. Predictive regression coefficients

Description: This table reports simulated regression coefficients for the predictive regressions summarized in Table 6. The reported results are from regressing future log excess equity returns (panel A), consumption growth (panel B), dividend growth (panel C), and real risk-free rates (panel D) on the current log price-dividend ratio. Regressions in the data are based on 1930–2008 annual historical data from the Bureau of Economic analysis and CRSP. Simulation regressions are based on 100,000 simulations with time periods equal to the historical data. The models are simulated monthly and then annualized for comparability with the historical data. For the simulations, the table reports the median price-dividend ratio regression coefficient and the % of simulated regression coefficients that are larger than the comparable regression coefficient in the data. Standard errors are Newey-West with 2*(horizon-1) lags.
Interpretation: Predictive regressions show the extent to which the price-dividend ratios predicts subsequent returns, consumption growth, dividend growth, and risk-free rates in the data and model.

	Data			Benchmark model $\hat{\beta}$		Extended model $\hat{\beta}$	
	$\hat{\beta}$	t	R^2	Median	% > data	Median	% > data
Panel A. Excess returns							
1 year	-0.09	-1.80	0.04	-0.04	74.7%	-0.04	80.9%
3 years	-0.26	-3.23	0.17	-0.13	74.9%	-0.12	80.7%
5 years	-0.41	-3.78	0.27	-0.21	74.6%	-0.20	80.4%
Panel B. Consumption growth							
1 year	0.01	1.59	0.06	0.00	9.9%	0.03	97.5%
3 years	0.01	0.59	0.01	0.00	33.6%	0.09	99.5%
5 years	0.00	-0.06	0.00	0.00	51.2%	0.13	99.5%
Panel C. Dividend growth							
1 year	0.07	1.98	0.09	0.01	0.5%	0.06	32.7%
3 years	0.11	1.33	0.06	0.01	5.7%	0.19	89.7%
5 years	0.09	1.21	0.04	0.01	19.8%	0.30	96.0%
Panel D. Risk-free rate							
1 year	0.01	1.15	0.03	-0.13	0.0%	-0.01	19.9%
3 years	0.03	0.82	0.03	-0.34	0.0%	-0.02	24.6%
5 years	0.05	1.06	0.05	-0.49	0.0%	-0.03	24.1%

Table IA.2. Vector autoregression results

Description: This table reports results from the vector autoregression (VAR) described by equation (IA.18). The nominal log yield on a one-month Treasury bill is $y1$. Inflation is one-month log seasonally-adjusted CPI inflation. Relative $y1$ and relative inflation are the difference between current yields and inflation and average values over the past twelve months. The yield spread between 10-year Treasury bonds and 3-month Treasury bills is $y120 - y3$. The excess return of the CRSP value weighted market return over the risk-free rate is $r_m - r_f$. The log dividend-price ratio, $d - p$, is calculated for the CRSP value-weighted market index using current prices and average dividends over the past twelve months. Results are for a 1-lag VAR of demeaned y1, inflation, relative $y1$, relative inflation, $r_m - r_f$, and $d - p$. Coefficients for dependent variables y1 and inflation are reported. The other dependent variables are omitted for brevity. Bootstrapped standard errors are in parentheses. * represents 10% significance, ** represents 5% significance, *** represents 1% significance.

Interpretation: Nominal interest rate shocks are highly persistent. Inflation shocks are less persistent and less predictable.

	1983–2012		1927–2012	
	$y1$	$inflation$	$y1$	$inflation$
Lagged Variables				
$y1$	0.9639***	0.1939*	0.9741***	0.0631
	(0.0202)	(0.1003)	(0.0116)	(0.0773)
$inflation$	0.0314	0.0737	0.0102*	0.7762***
	(0.0297)	(0.1734)	(0.0062)	(0.0709)
$relative\ y1$	-0.0976**	0.1295	-0.1752***	0.5909***
	(0.0457)	(0.1585)	(0.0407)	(0.1599)
$relative\ inflation$	-0.0136	0.3268*	-0.003	-0.4554***
	(0.0281)	(0.1767)	(0.0056)	(0.0837)
$y120 - y3$	-0.0032	-0.002	-0.0062**	0.0014
	(0.0036)	(0.0155)	(0.0024)	(0.0122)
$r_m - r_f$	0.0013*	0.0083*	0.0008**	0.0061*
	(0.0007)	(0.0042)	(0.0004)	(0.0034)
$d - p$	0.0001	0.0002	0.0000	-0.0002
	(0.0001)	(0.0005)	(0.0000)	(0.0003)
R^2	0.95	0.24	0.95	0.32

Table IA.3. Valuation risk pricing in the cross section of stock returns

Description: Value-weighted decile portfolios are formed at the end of each month by sorting stocks based on covariance with risk-free rate news over the past three years. The table reports betas with respect to risk free rate news, average size, and average excess returns for each portfolio. The table also reports results for time series regressions of excess returns on excess market returns (the CAPM regression) and excess market returns (*rmrf*), the Fama-French size factor (*smb*), and the Fama-French value factor (*hml*) (the 3 Factor regression). The sample is NYSE, AMEX, and NASDAQ common stocks. Standard errors for the 10-1 portfolio difference are reported in parentheses. * represents 10% significance, ** represents 5% significance, *** represents 1% significance.

Interpretation: Expected returns do not vary significantly across the decile portfolios.

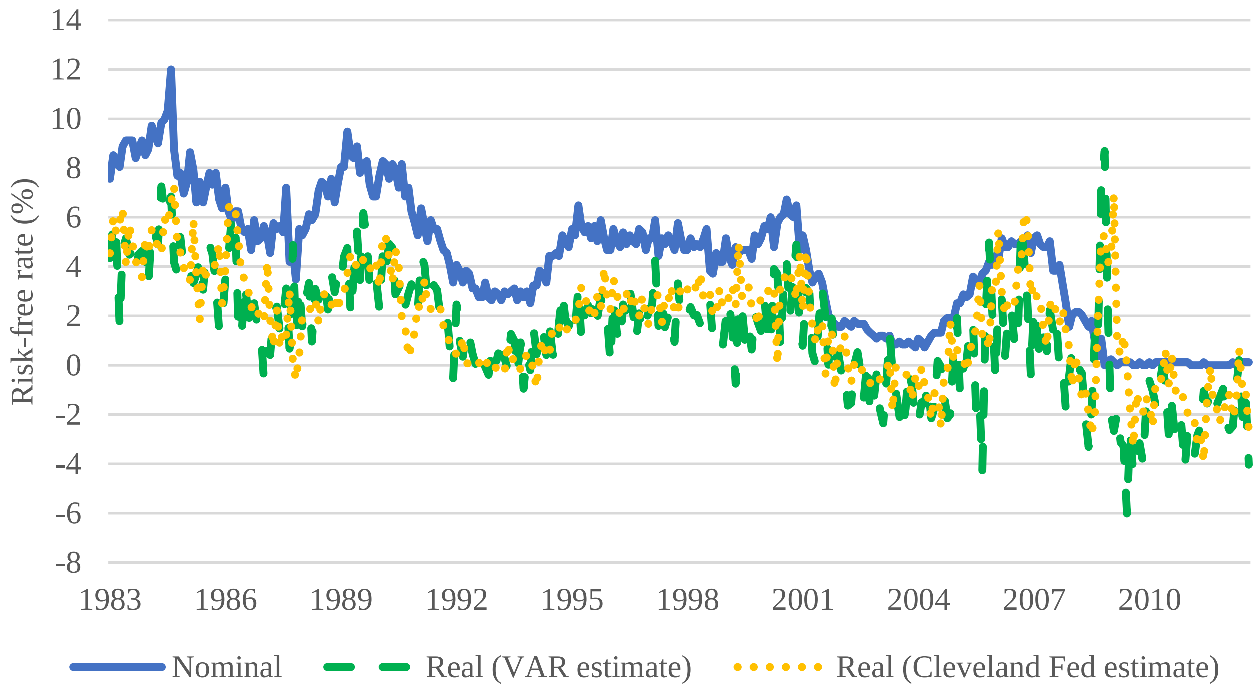
Panel A. 1985–2012

	r_f News	Market Cap	Excess	CAPM	3 Factor	Factor Loadings (Betas)		
Decile	Beta	($B)	Return	Alpha	Alpha	*rmrf*	*smb*	*hml*
1	-0.17	0.72	0.63%	-0.19%	-0.16%	1.27	0.61	-0.06
2	0.07	1.36	0.94%	0.24%	0.30%	1.10	0.22	-0.15
3	-0.04	1.94	0.87%	0.25%	0.23%	1.04	0.07	0.04
4	0.13	2.42	0.65%	0.06%	0.03%	1.00	-0.04	0.09
5	0.00	2.74	0.51%	-0.03%	-0.05%	0.94	-0.10	0.03
6	0.02	2.76	0.48%	-0.06%	-0.08%	0.93	-0.14	0.05
7	0.03	2.58	0.54%	-0.02%	-0.04%	0.97	-0.11	0.03
8	0.15	2.21	0.68%	0.06%	0.08%	1.04	-0.13	-0.07
9	0.14	1.69	0.61%	-0.06%	-0.04%	1.10	0.01	-0.06
10	0.41	0.85	0.21%	-0.62%	-0.44%	1.21	0.55	-0.47
10-1	0.58**	0.13**	-0.42%	-0.42%	-0.27%	-0.06	-0.07	-0.41***
	(0.23)	(0.06)	(0.33%)	(0.34%)	(0.34%)	(0.08)	(0.11)	(0.12)

Panel B. 1929–2012

	r_f News	Market Cap	Excess	CAPM	3 Factor	Factor Loadings (Betas)		
Decile	Beta	($B)	Return	Alpha	Alpha	*rmrf*	*smb*	*hml*
1	-0.01	0.17	0.66%	-0.05%	-0.12%	1.15	0.52	-0.03
2	0.00	0.48	0.66%	0.04%	0.03%	1.04	0.20	-0.06
3	0.03	0.69	0.70%	0.13%	0.12%	0.99	0.08	-0.01
4	0.06	0.86	0.71%	0.15%	0.15%	0.96	0.02	0.00
5	0.01	0.98	0.60%	0.04%	0.02%	0.97	-0.03	0.06
6	0.03	1.05	0.56%	-0.01%	-0.03%	0.98	-0.03	0.09
7	0.06	1.08	0.58%	-0.01%	-0.02%	1.03	-0.08	0.08
8	0.06	1.05	0.56%	-0.07%	-0.10%	1.08	0.00	0.11
9	0.10	0.83	0.61%	-0.07%	-0.12%	1.15	0.04	0.17
10	0.11	0.38	0.58%	-0.18%	-0.27%	1.23	0.50	0.03
10-1	0.13	0.21***	-0.09%	-0.13%	-0.14%	0.07**	-0.02	0.05
	(0.09)	(0.02)	(0.18%)	(0.18%)	(0.18%)	(0.03)	(0.06)	(0.05)

Figure IA.1. Risk-free rate, 1983–2012



Description: This figure plots the monthly nominal and real risk-free rate from 1983 to 2012. The nominal risk-free rate is the yield on a one-month nominal treasury bill. The real risk-free rate is estimated using VAR analysis. For comparison purposes, the Federal Reserve Bank of Cleveland's real risk-free rate estimate is also plotted.
Interpretation: Real risk-free rate estimates from the VAR model closely track estimates from the Federal Reserve Bank of Cleveland and generally follow the same pattern as nominal interest rates.